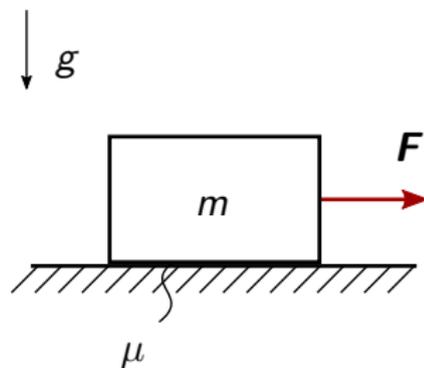


# Reibung und Seilreibung

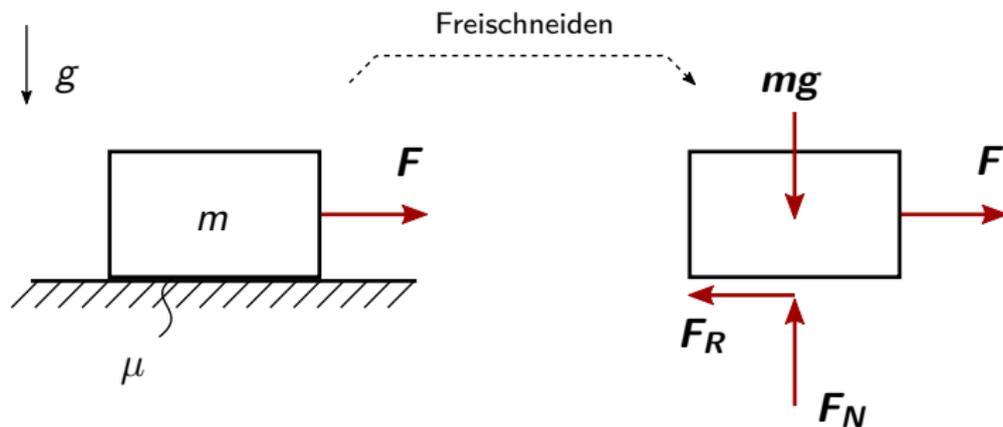
Dominik Zobel

- Haftreibung
- Reibkegel
- Beispiel Reibblock
- Seilreibung
- Aufgabe Seilreibung
- Aufgabe Reibung

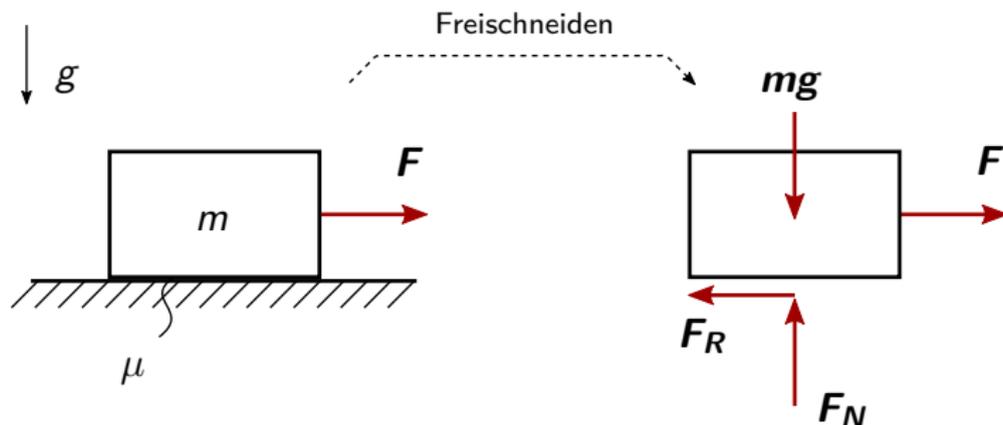
# Haftreibung Block



# Haftreibung Block

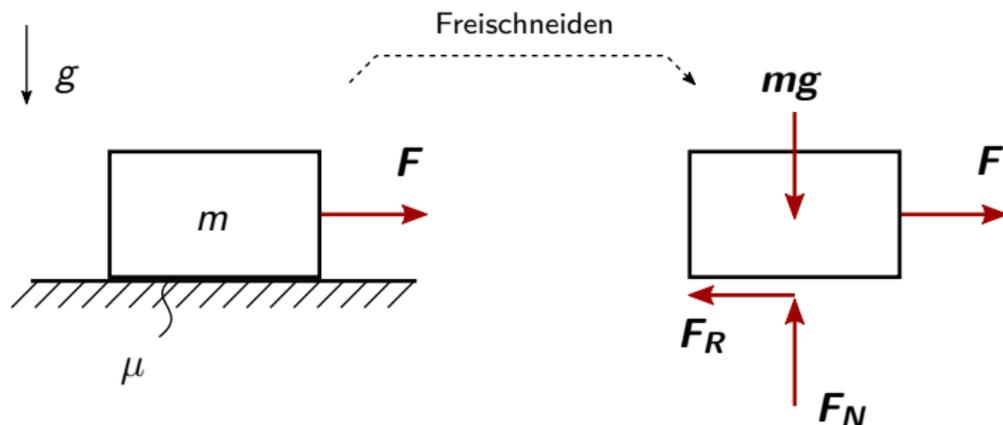


# Haftreibung Block



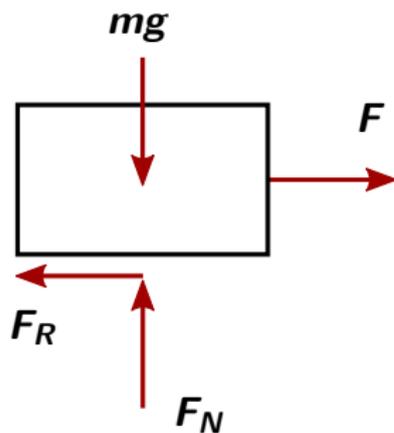
$$|F_R| \leq \mu |F_N|$$

# Haftreibung Block



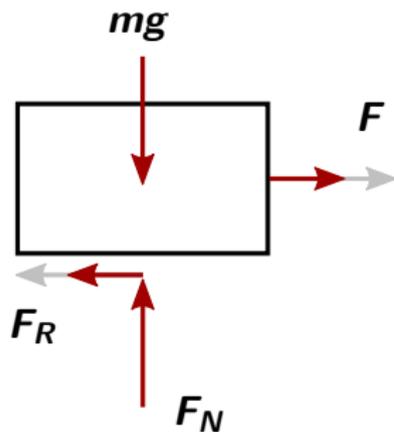
$$|F_R| \leq \mu |F_N|$$

$$|F_{R,\max}| = \mu |F_N|$$



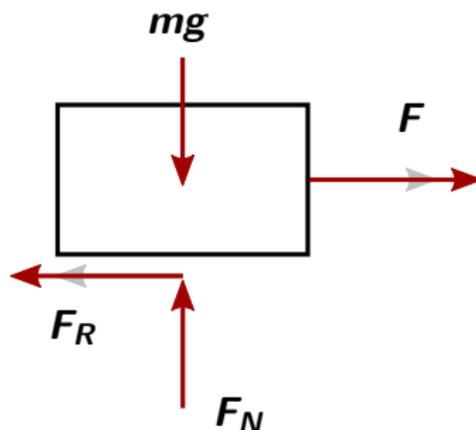
$$|F_R| \leq \mu |F_N|$$

$$|F_{R,\max}| = \mu |F_N|$$



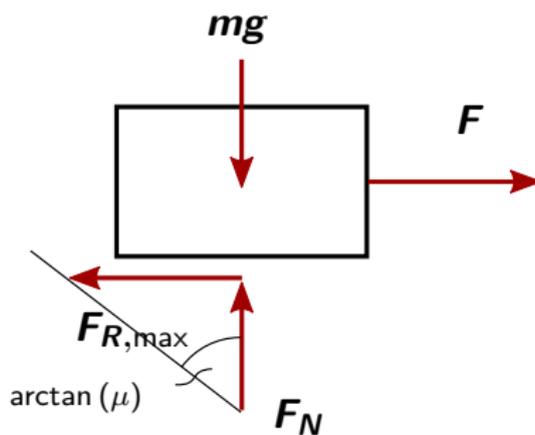
$$|F_R| \leq \mu |F_N|$$

$$|F_{R,\max}| = \mu |F_N|$$



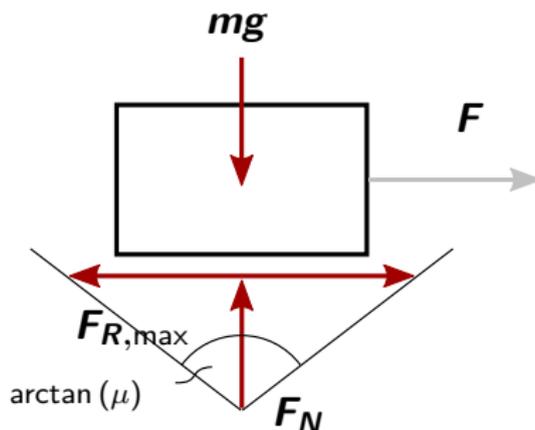
$$|F_R| \leq \mu |F_N|$$

$$|F_{R,\max}| = \mu |F_N|$$



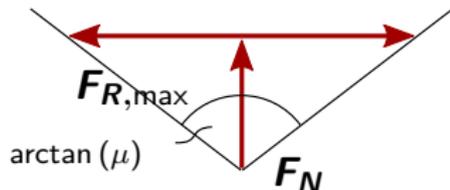
$$|\mathbf{F}_R| \leq \mu |\mathbf{F}_N|$$

$$|\mathbf{F}_{R,max}| = \mu |\mathbf{F}_N|$$



$$|\mathbf{F}_R| \leq \mu |\mathbf{F}_N|$$

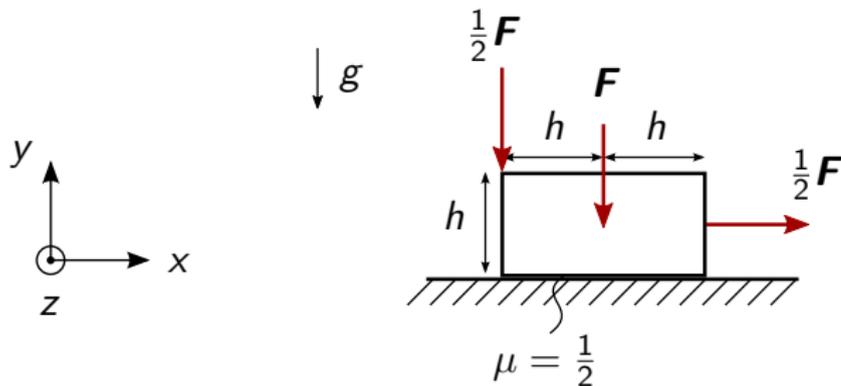
$$|\mathbf{F}_{R,max}| = \mu |\mathbf{F}_N|$$



$\mu = \frac{F_{R,max}}{F_N}$  ist die Steigung des Reibkegels

## Beispiel zum Reibblock

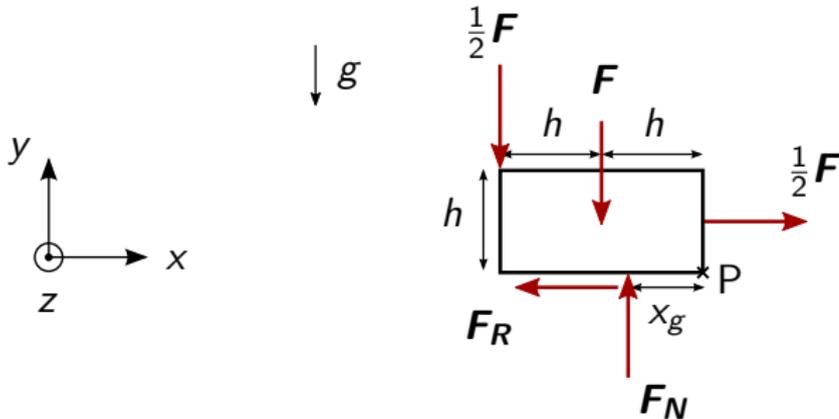
Gegeben:  $g, h, F, \mu$



- Freikörperbild zeichnen
- Betrag und Angriffspunkt der Normalkraft  $F_N$  bestimmen

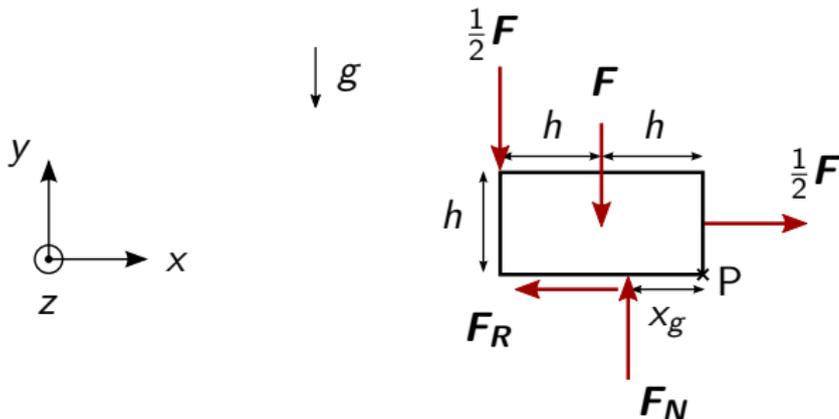
# Beispiel zum Reibblock

Gegeben:  $g, h, F, \mu$



## Beispiel zum Reibblock

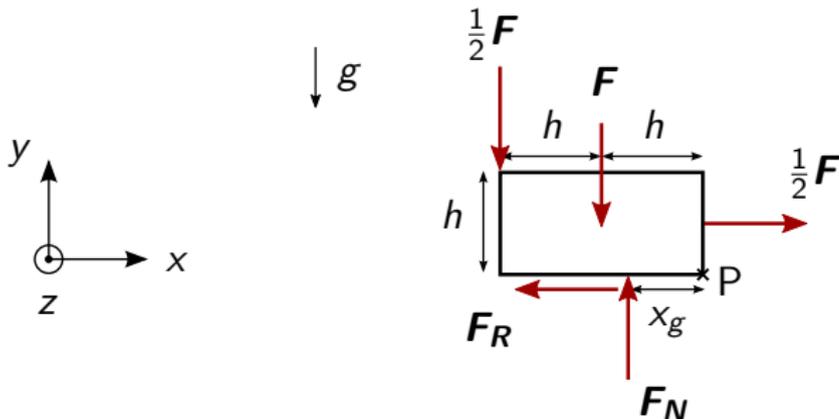
Gegeben:  $g, h, F, \mu$



$$\sum M^P : 0 = -x_g F_N - \frac{h}{2} \frac{1}{2} F + hF + 2h \frac{1}{2} F$$

## Beispiel zum Reibblock

Gegeben:  $g, h, F, \mu$

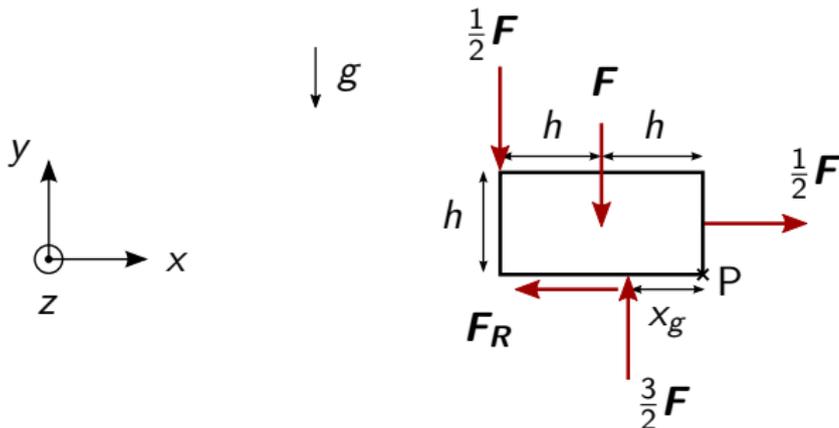


$$\sum M^P : 0 = -x_g F_N - \frac{h}{2} \frac{1}{2} F + hF + 2h \frac{1}{2} F$$

$$\sum F_y : 0 = -F - \frac{1}{2} F + F_N \Rightarrow F_N = \frac{3}{2} F$$

## Beispiel zum Reibblock

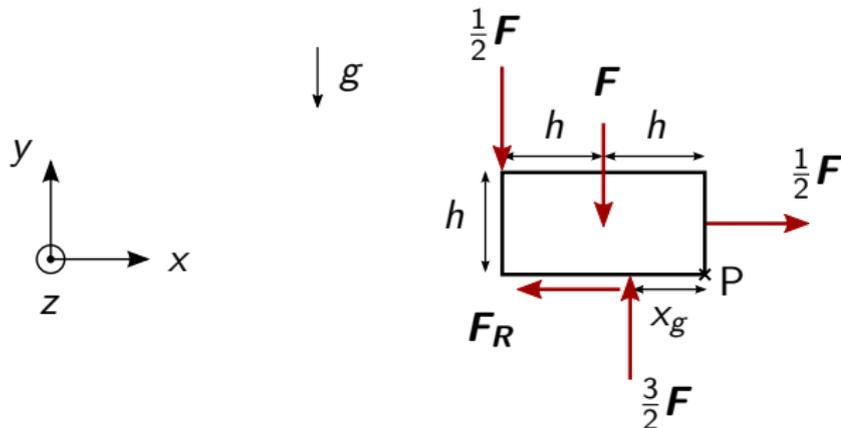
Gegeben:  $g, h, F, \mu$



$$\sum M^P : 0 = -x_g F_N - \frac{h}{2} \frac{1}{2} F + hF + 2h \frac{1}{2} F$$

## Beispiel zum Reibblock

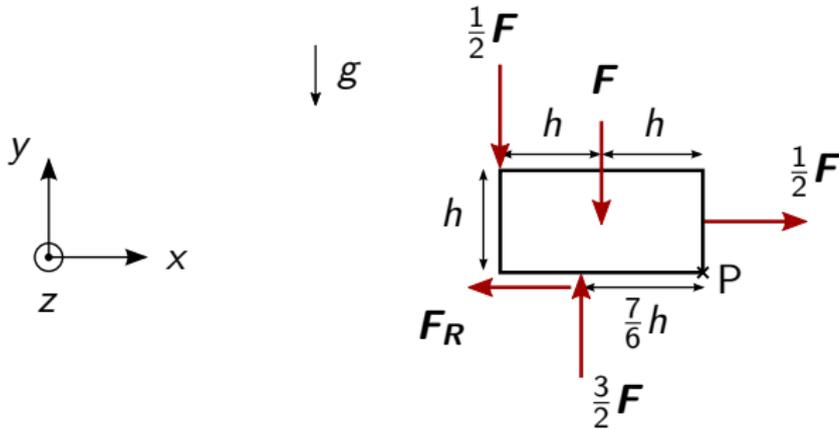
Gegeben:  $g, h, F, \mu$



$$\sum M^P : 0 = -x_g F_N - \frac{h}{2} \frac{1}{2} F + hF + 2h \frac{1}{2} F$$
$$0 = -x_g \frac{3}{2} F + \frac{7}{4} hF \Rightarrow x_g = \frac{7}{6} h$$

# Beispiel zum Reibblock

Gegeben:  $g, h, F, \mu$

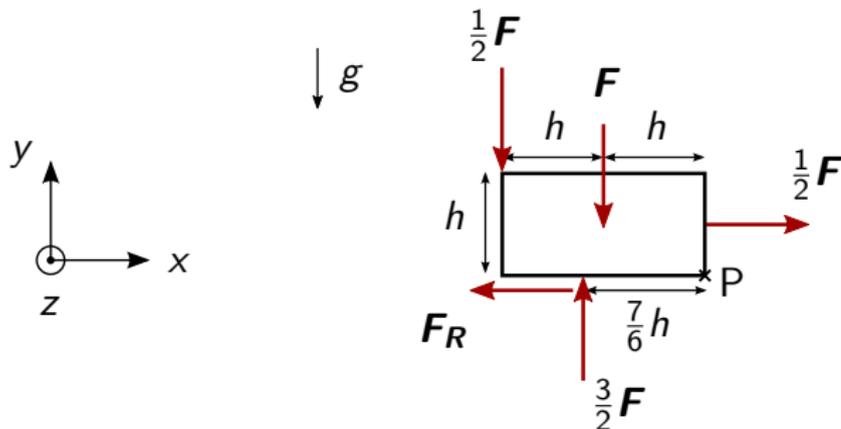


$$|\mathbf{F}_R| \leq \mu |\mathbf{F}_N|$$

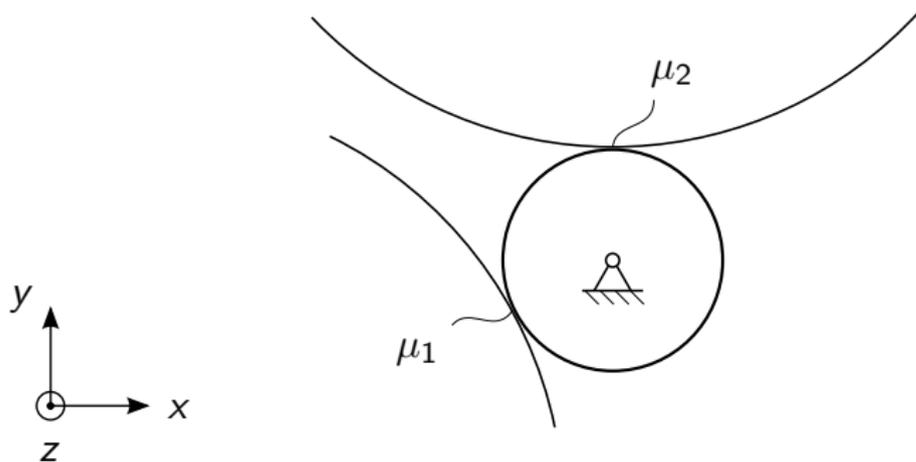
$$|\mathbf{F}_{R,\max}| = \mu |\mathbf{F}_N|$$

## Beispiel zum Reibblock

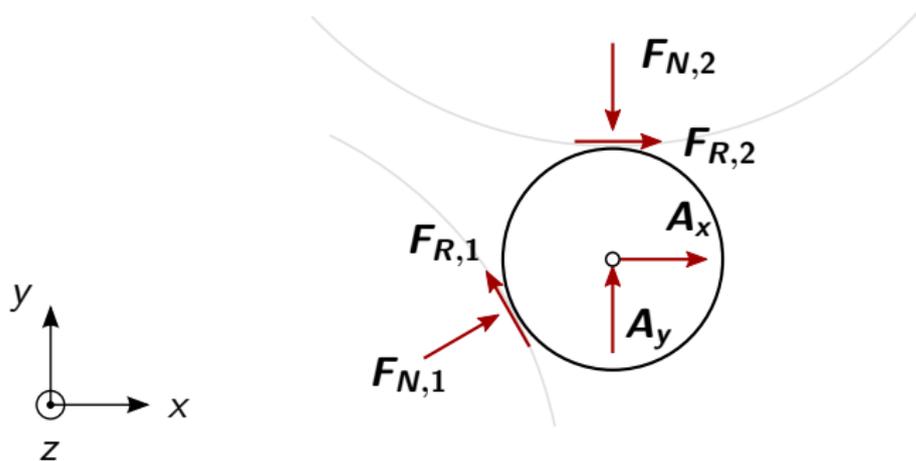
Gegeben:  $g, h, F, \mu$

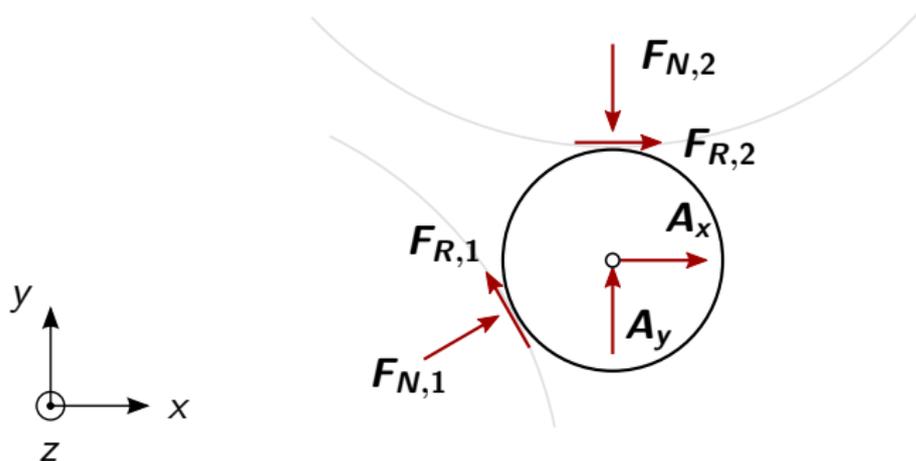


$$|F_R| \leq \mu |F_N| \qquad |F_{R,\max}| = \mu |F_N|$$
$$F_N = \frac{3}{2}F \quad \Rightarrow \quad F_{R,\max} = \frac{3}{4}F \quad , \quad F_R = \frac{1}{2}F$$

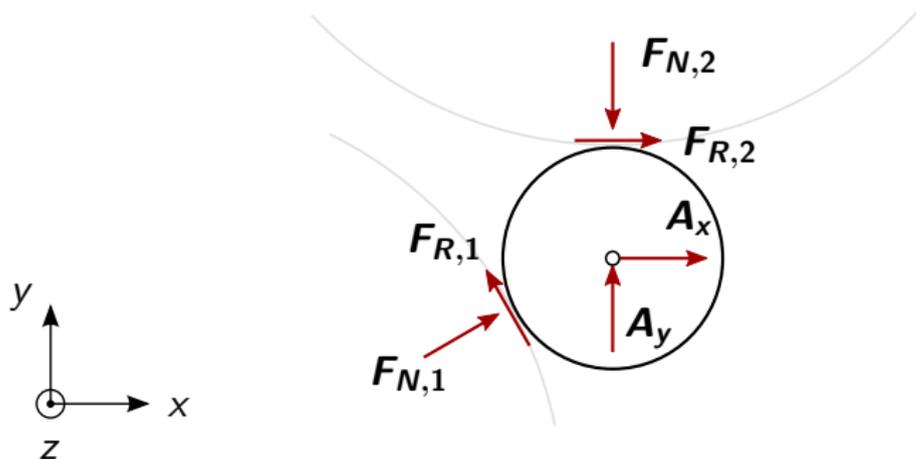


# Reibzylinder





$F_R$  entgegen Bewegungsrichtung annehmen



$F_R$  entgegen Bewegungsrichtung annehmen

Bei Zylindern  $F_{R,i}$  in gleiche Drehrichtung annehmen

Gleitreibung findet statt, wenn Haftreibung überwunden ist

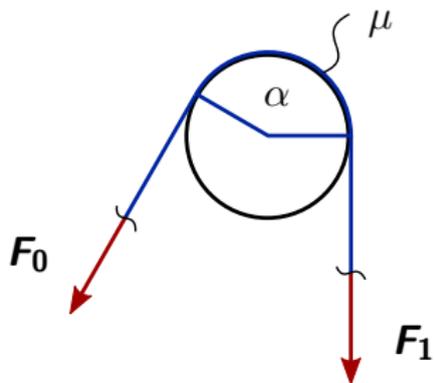
Gleitreibung findet statt, wenn Haftreibung überwunden ist

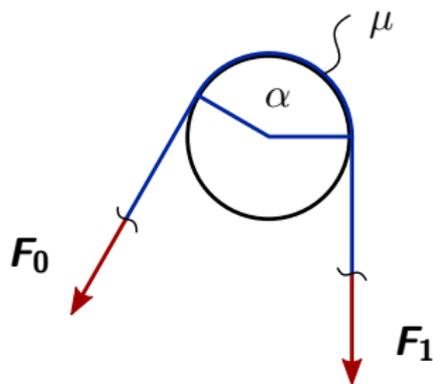
$$|\mathbf{F}_R| = \mu_G |\mathbf{F}_N|$$

Gleitreibung findet statt, wenn Haftreibung überwunden ist

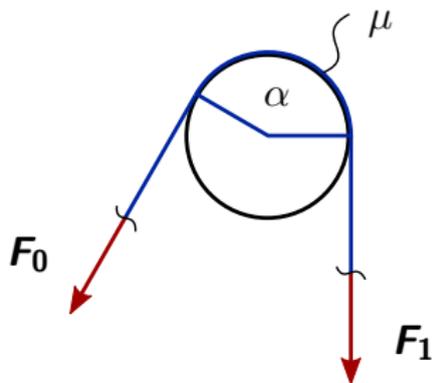
$$|\mathbf{F}_R| = \mu_G |\mathbf{F}_N|$$

$$\mu_G < \mu(H)$$



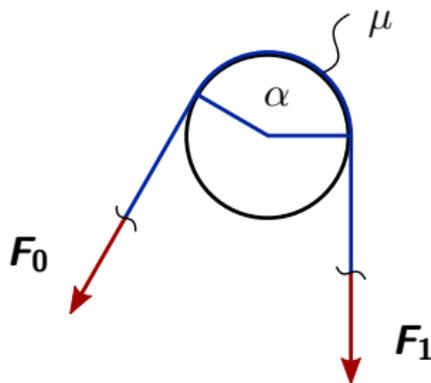


Eytelweinsche Gleichung  $\frac{|F_1|}{|F_0|} = e^{\mu\alpha}$



Eytelweinsche Gleichung  $\frac{|F_1|}{|F_0|} = e^{\mu\alpha}$

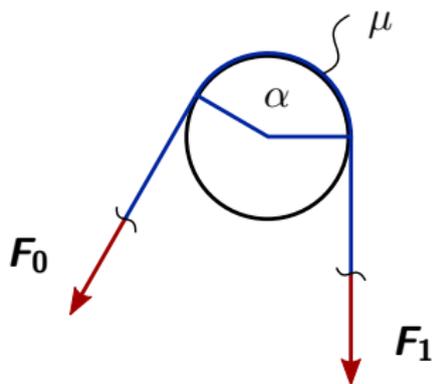
Es gilt  $e^{-x} = \frac{1}{e^x}$  und  $e^0 = 1$



Eytelweinsche Gleichung  $\frac{|F_1|}{|F_0|} = e^{\mu\alpha}$

Es gilt  $e^{-x} = \frac{1}{e^x}$  und  $e^0 = 1$

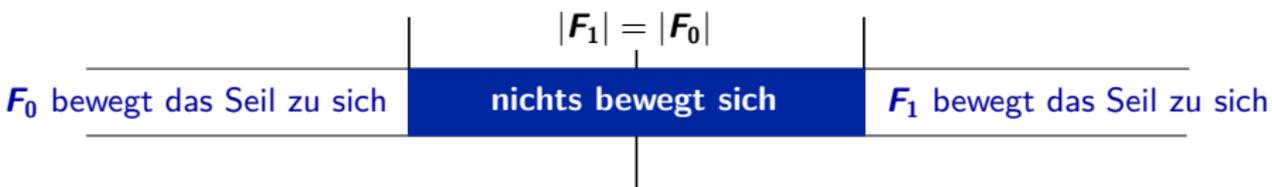
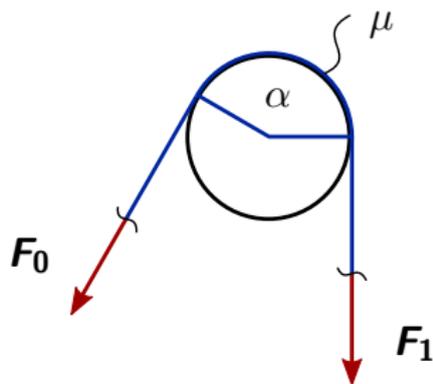
Dicke des Zylinders ist egal, nur Aufgewinkel von Bedeutung

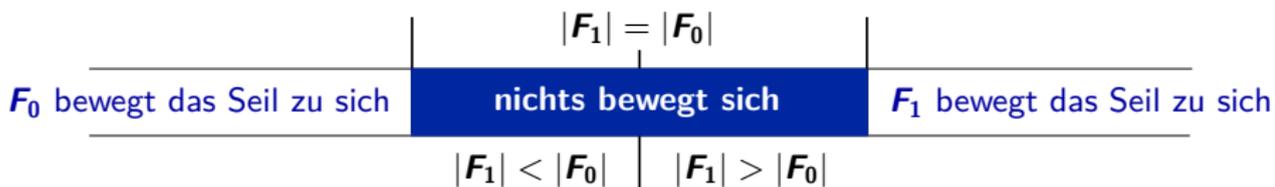
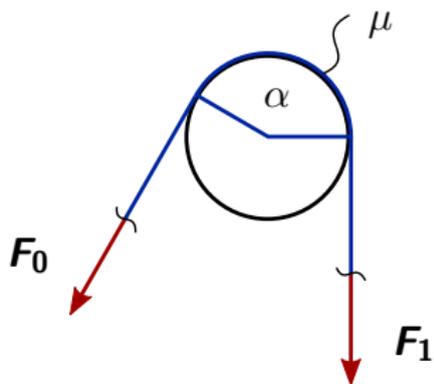


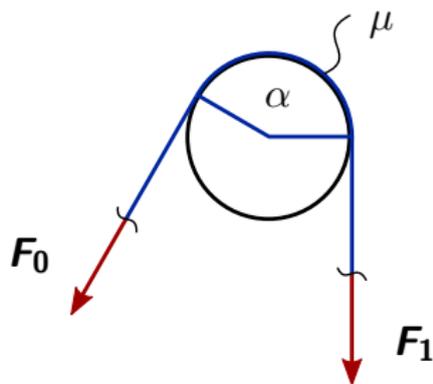
$F_0$  bewegt das Seil zu sich

nichts bewegt sich

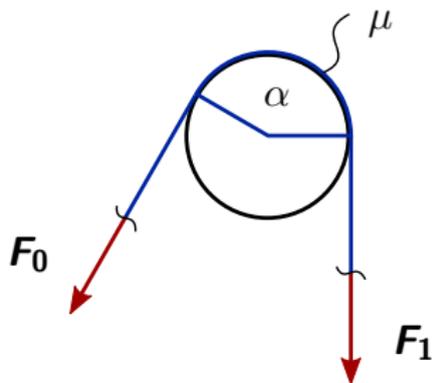
$F_1$  bewegt das Seil zu sich





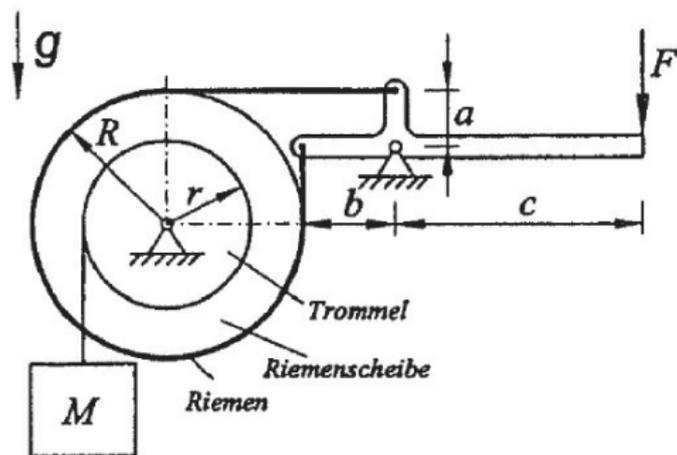


	$ F_1  =  F_0 $	$ F_1  > e^{\mu\alpha}  F_0 $
$F_0$ bewegt das Seil zu sich	nichts bewegt sich	
	$ F_1  <  F_0 $	$ F_1  >  F_0 $
	$ F_1  = e^{\mu\alpha}  F_0 $	
		$F_1$ bewegt das Seil zu sich

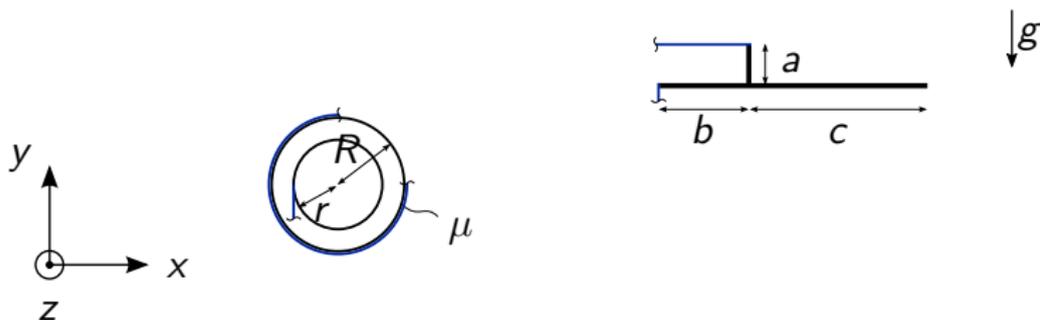


$ F_1  < e^{-\mu\alpha}  F_0 $	$ F_1  =  F_0 $	$ F_1  > e^{\mu\alpha}  F_0 $
$F_0$ bewegt das Seil zu sich	<b>nichts bewegt sich</b>	$F_1$ bewegt das Seil zu sich
$ F_1  = e^{-\mu\alpha}  F_0 $	$ F_1  <  F_0 $ $ F_1  >  F_0 $	$ F_1  = e^{\mu\alpha}  F_0 $

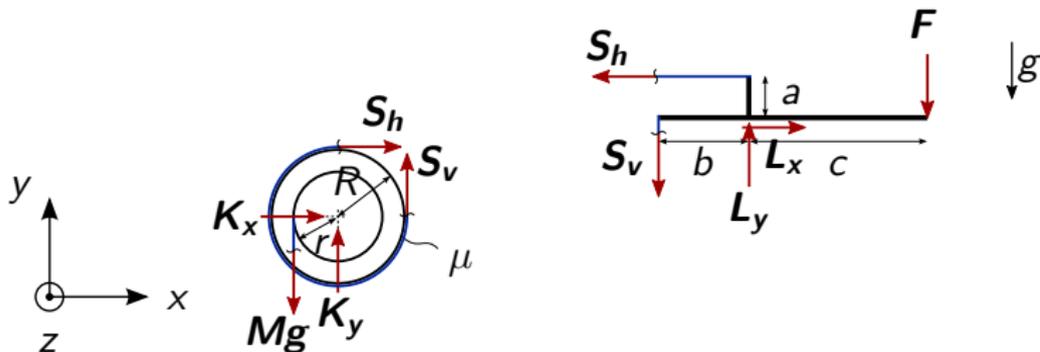
## H06 – Aufgabe 4



## a) Freikörperbild

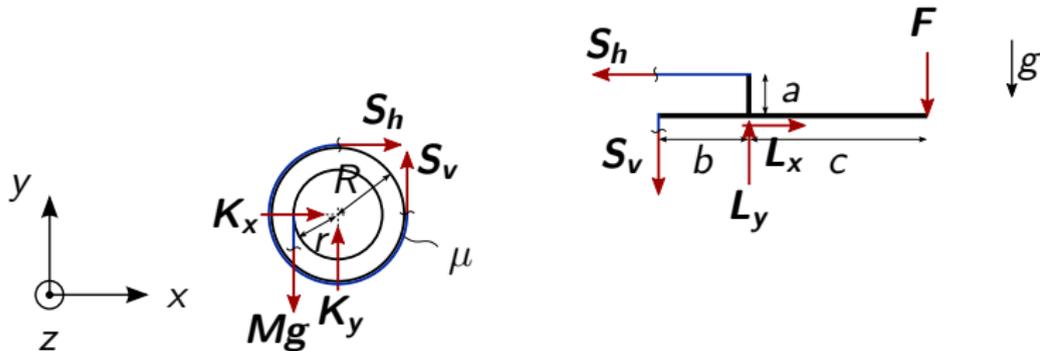
Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$ 

## a) Freikörperbild

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$ 

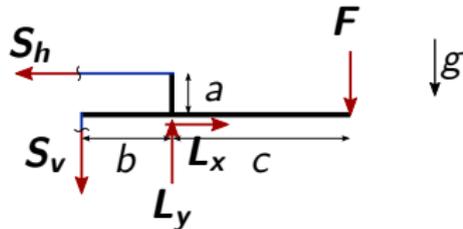
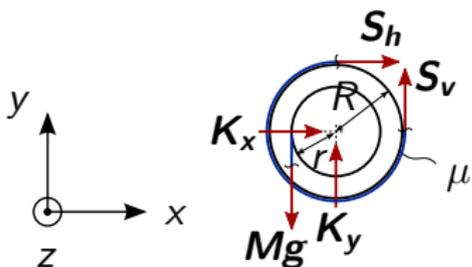
b) Erforderliche Gleichungen, um  $F$  zu bestimmen

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



b) Erforderliche Gleichungen, um  $\mathbf{F}$  zu bestimmen

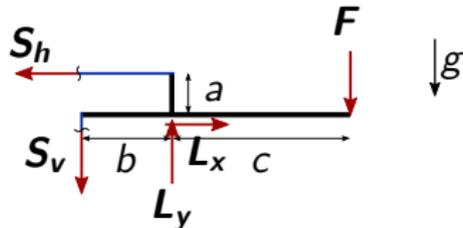
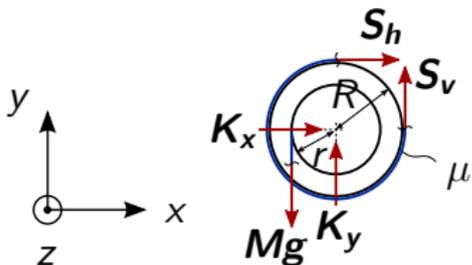
Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



$$\sum M_z^K : 0 = rMg - RS_h + RS_v$$

b) Erforderliche Gleichungen, um  $\mathbf{F}$  zu bestimmen

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$

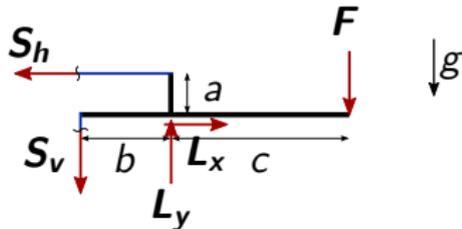
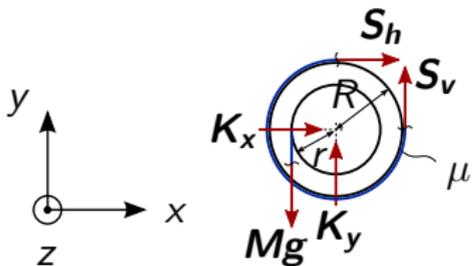


$$\sum M_z^K : 0 = rMg - RS_h + RS_v$$

$$S_h = S_v + \frac{r}{R}Mg$$

b) Erforderliche Gleichungen, um  $\mathbf{F}$  zu bestimmen

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



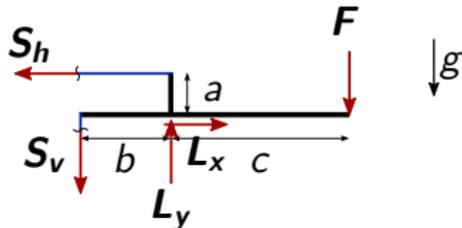
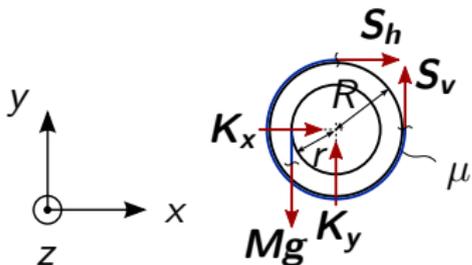
$$\sum M_z^K : 0 = rMg - RS_h + RS_v$$

$$S_h = S_v + \frac{r}{R}Mg$$

$$S_h = S_v e^{\frac{3}{2}\pi\mu}$$

b) Erforderliche Gleichungen, um  $F$  zu bestimmen

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



$$\sum M_z^K : 0 = rMg - RS_h + RS_v$$

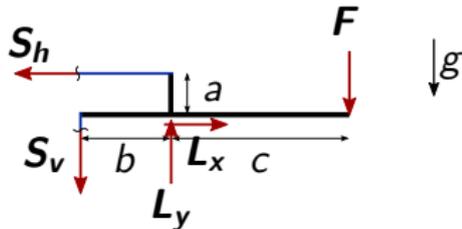
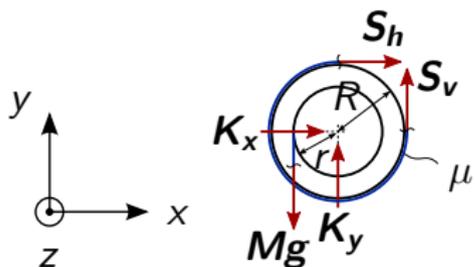
$$S_h = S_v + \frac{r}{R}Mg$$

$$S_h = S_v e^{\frac{3}{2}\pi\mu}$$

$$\sum M_z^L : 0 = bS_v + aS_h - cF$$

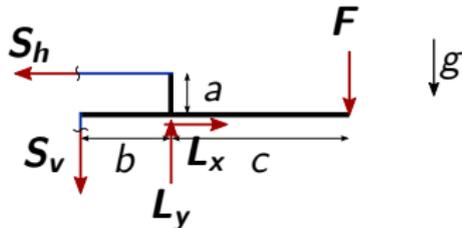
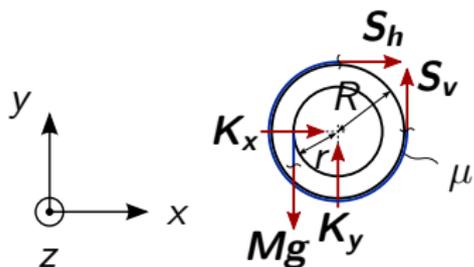
c)  $\mathbf{F}$  in gegebenen Größen angeben

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



c)  $\mathbf{F}$  in gegebenen Größen angeben

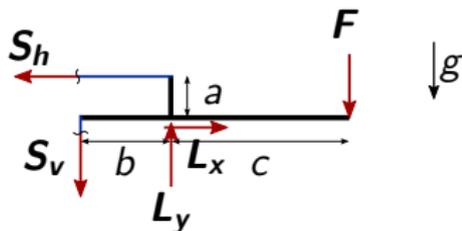
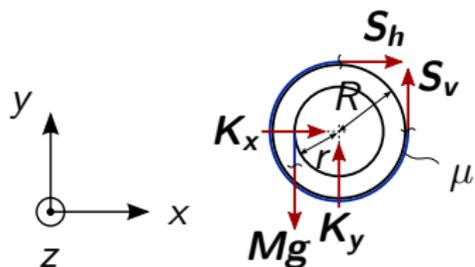
Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



$$S_v = \frac{r}{R} \cdot \frac{Mg}{e^{\mu \frac{3}{2}\pi} - 1}$$

c)  $F$  in gegebenen Größen angeben

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$

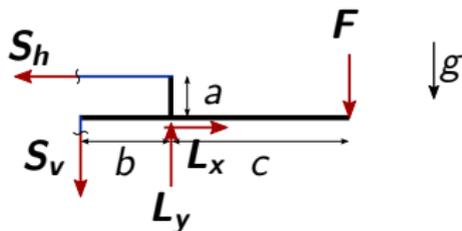
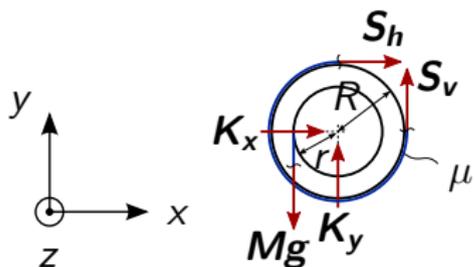


$$S_v = \frac{r}{R} \cdot \frac{Mg}{e^{\mu \frac{3}{2}\pi} - 1}$$

$$S_h = \frac{r}{R} \cdot \frac{Mg e^{\frac{3}{2}\pi\mu}}{e^{\frac{3}{2}\pi\mu} - 1}$$

c)  $F$  in gegebenen Größen angeben

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



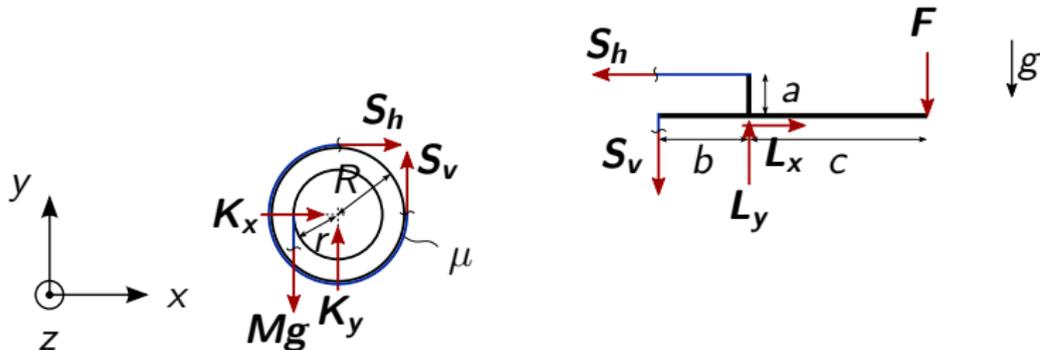
$$S_v = \frac{r}{R} \cdot \frac{Mg}{e^{\mu \frac{3}{2}\pi} - 1}$$

$$S_h = \frac{r}{R} \cdot \frac{Mg e^{\frac{3}{2}\pi\mu}}{e^{\frac{3}{2}\pi\mu} - 1}$$

$$F = \frac{r}{cR} \cdot \frac{Mg}{e^{\frac{3}{2}\pi\mu} - 1} \left( b + a e^{\frac{3}{2}\pi\mu} \right)$$

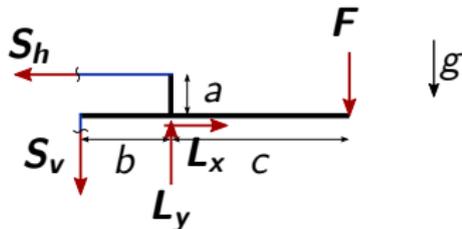
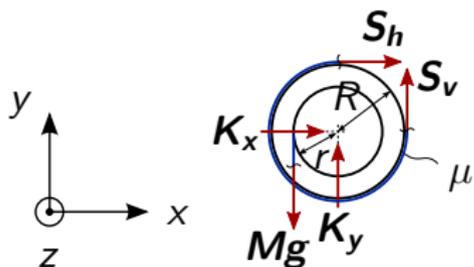
d)  $F$  berechnen, wenn  $Mg$  auf der anderen Seite angreift

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$



d)  $F$  berechnen, wenn  $Mg$  auf der anderen Seite angreift

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$

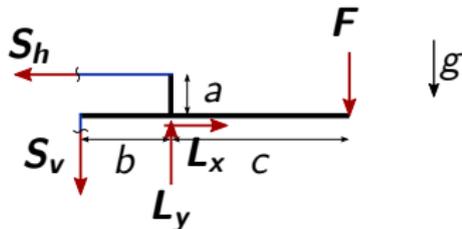
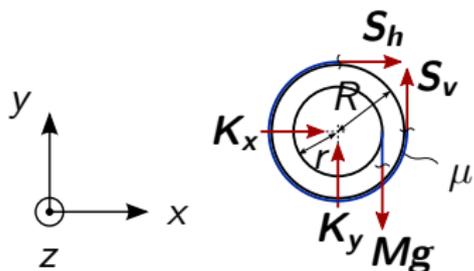


$$\sum M_z^K : 0 = +rMg - RS_h + RS_v$$

$$S_h = S_v e^{+\frac{3}{2}\pi\mu}$$

d)  $F$  berechnen, wenn  $Mg$  auf der anderen Seite angreift

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$

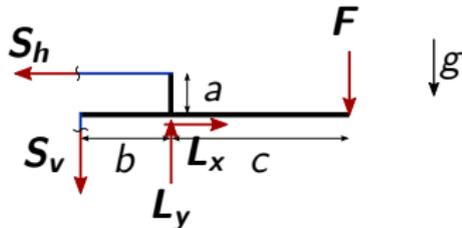
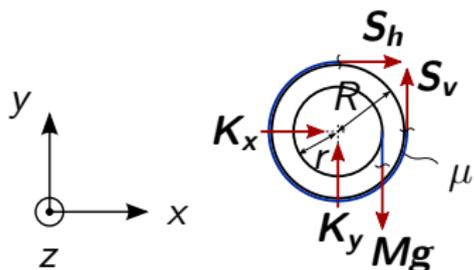


$$\sum M_z^K : 0 = -rMg - RS_h + RS_v$$

$$S_h = S_v e^{-\frac{3}{2}\pi\mu}$$

d)  $F$  berechnen, wenn  $Mg$  auf der anderen Seite angreift

Gegeben:  $g$ ;  $\mu$ ;  $a$ ;  $b$ ;  $c$ ;  $r$ ;  $R$ ;  $M$

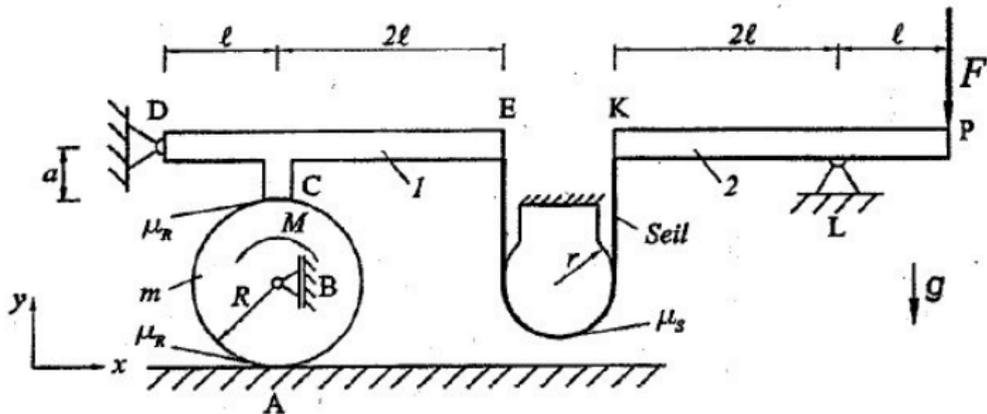


$$\sum M_z^K : 0 = -rMg - RS_h + RS_v$$

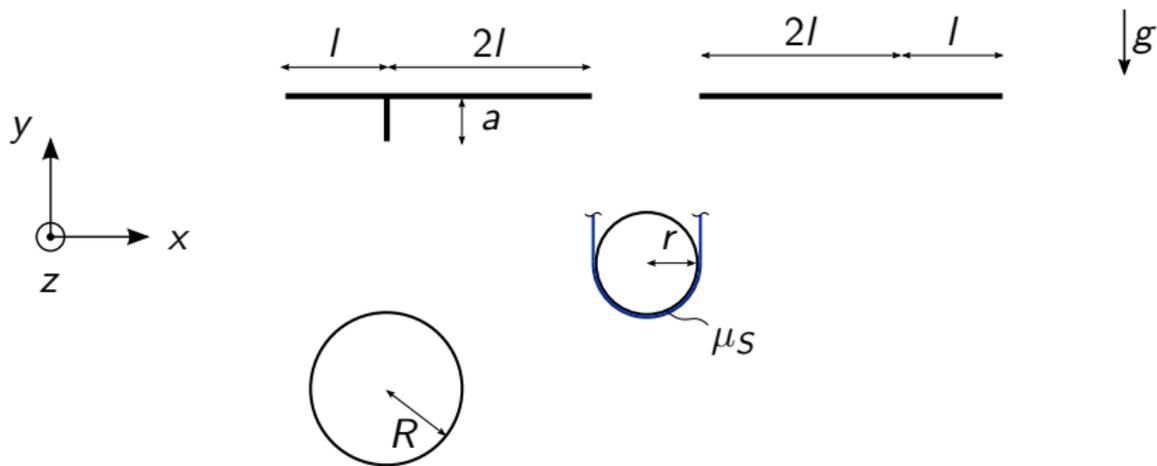
$$S_h = S_v e^{-\frac{3}{2}\pi\mu}$$

$$F = \frac{r}{cR} \cdot \frac{Mg}{e^{-\frac{3}{2}\pi\mu} - 1} \left( b + ae^{-\frac{3}{2}\pi\mu} \right)$$

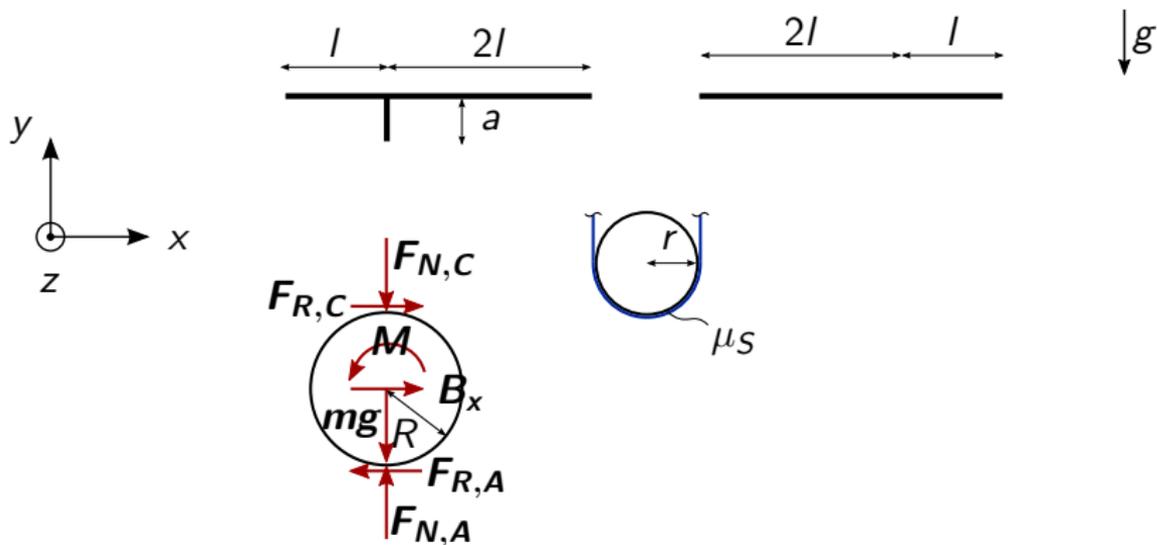
## H07 – Aufgabe 3



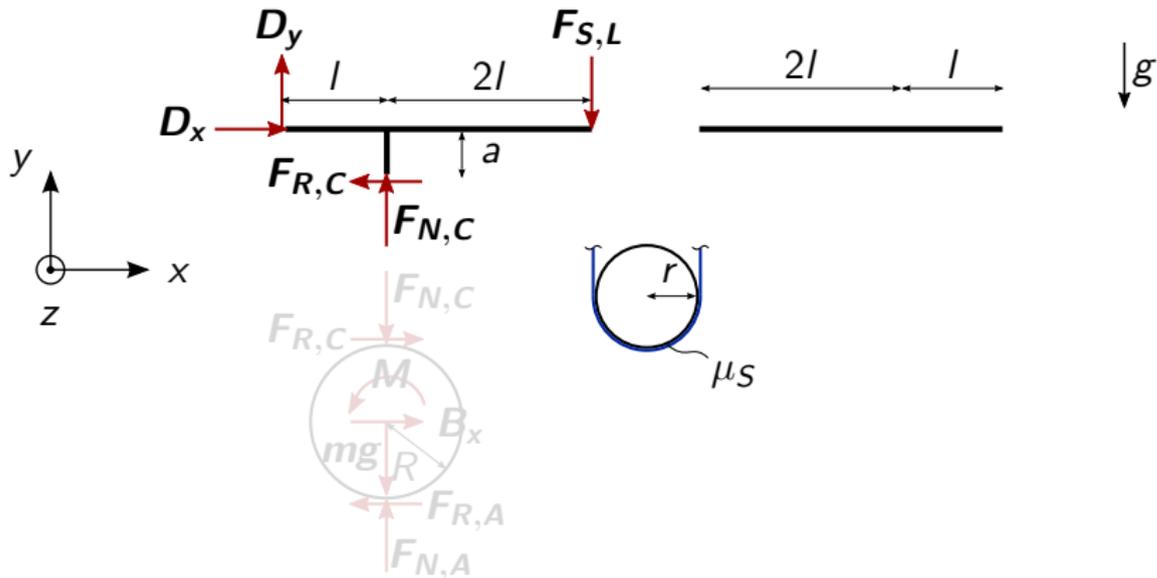
## a) Freikörperbild

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$ 

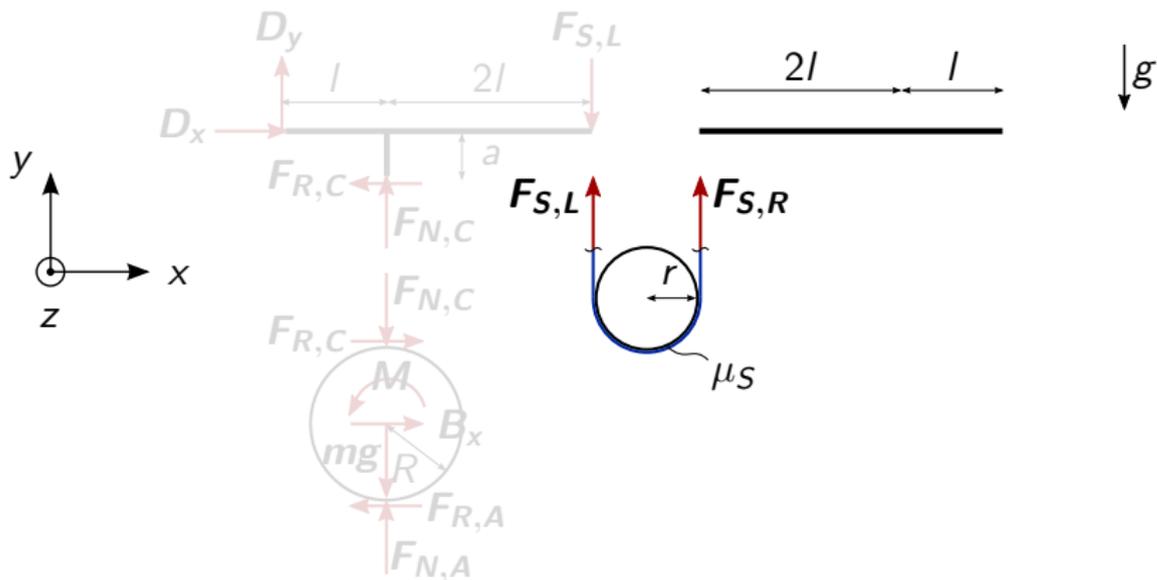
## a) Freikörperbild

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$ 

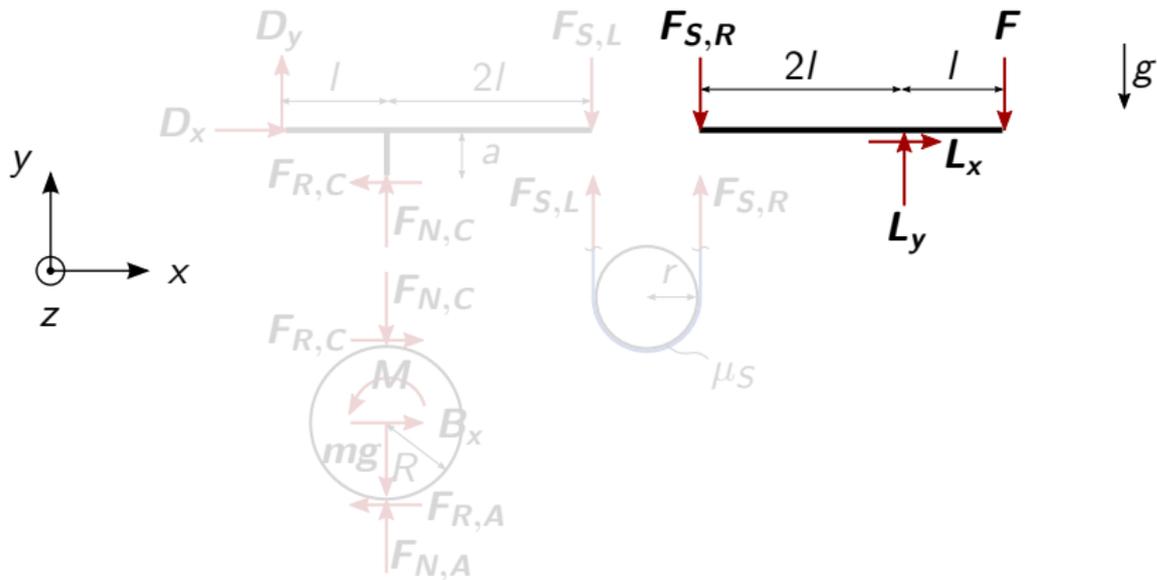
## a) Freikörperbild

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$ 

## a) Freikörperbild

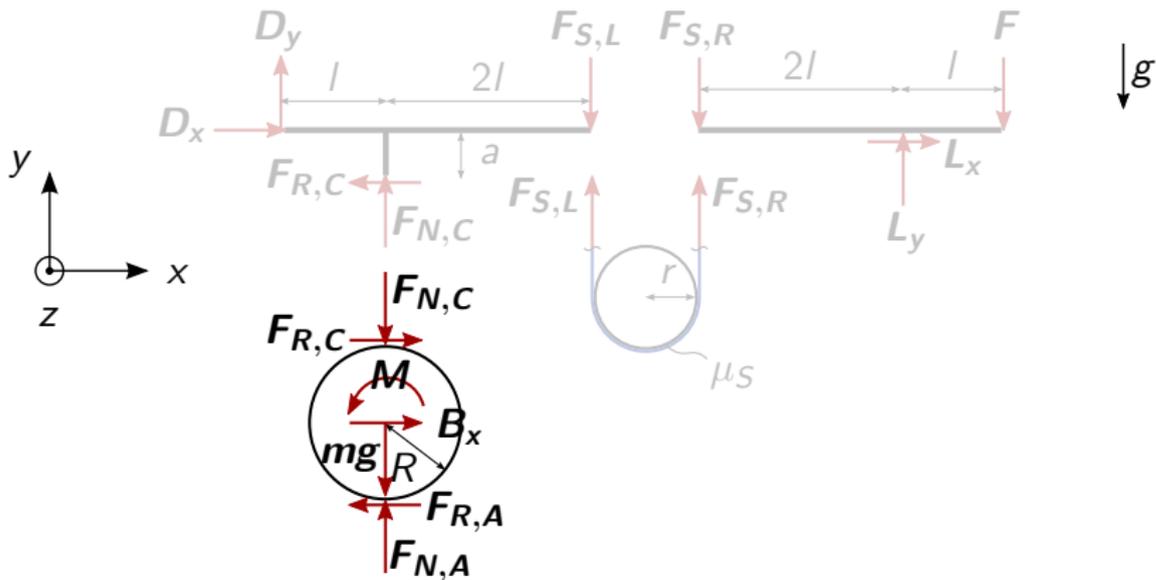
Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$ 

## a) Freikörperbild

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$ 

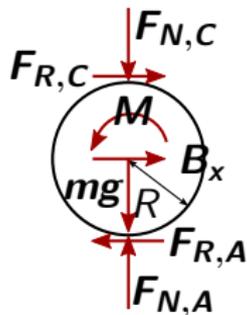
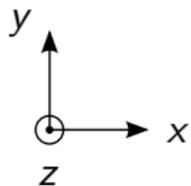
b)  $F_{N,C}$  bestimmen, damit das Rad ruht

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



b)  $F_{N,C}$  bestimmen, damit das Rad ruht

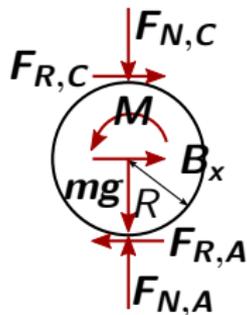
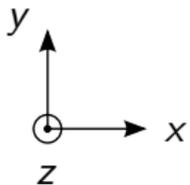
Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\sum F_y : 0 = -mg - F_{N,C} + F_{N,A}$$

b)  $F_{N,C}$  bestimmen, damit das Rad ruht

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$

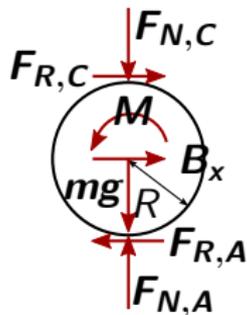
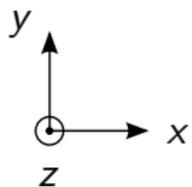


$$\sum F_y : 0 = -mg - F_{N,C} + F_{N,A}$$

$$\sum M_z^B : 0 = M - RF_{R,C} - RF_{R,A}$$

b)  $F_{N,C}$  bestimmen, damit das Rad ruht

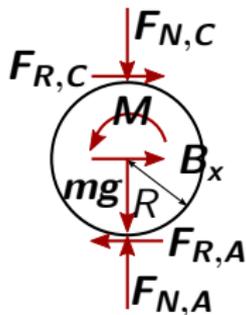
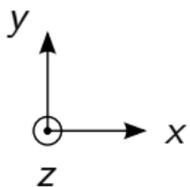
Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\begin{aligned} \sum F_y : 0 &= -mg - F_{N,C} + F_{N,A} \\ \sum M_z^B : 0 &= M - RF_{R,C} - RF_{R,A} \\ \Rightarrow F_{N,C} &= \frac{M - R\mu_R mg}{2R\mu_R} \end{aligned}$$

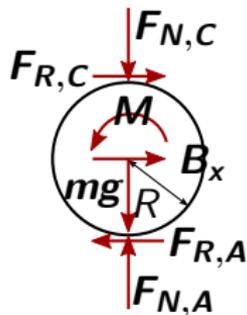
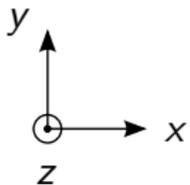
c) Lagerreaktionen in Punkt  $B$  bestimmen,  $F_{N,C}$  sei bekannt

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



c) Lagerreaktionen in Punkt  $B$  bestimmen,  $F_{N,C}$  sei bekannt

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$

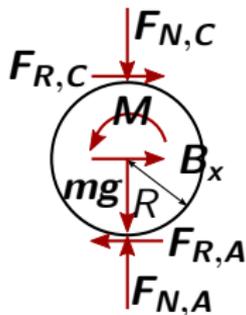
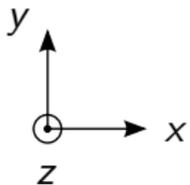


$$\sum F_x : 0 = B_x + F_{R,C} - F_{R,A}$$



c) Lagerreaktionen in Punkt  $B$  bestimmen,  $F_{N,C}$  sei bekannt

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$

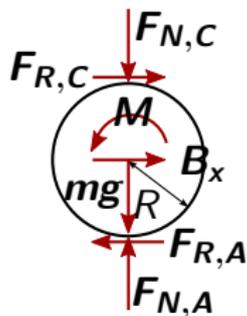
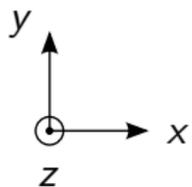


$$\sum F_x : 0 = B_x + F_{R,C} - F_{R,A}$$

$$\sum F_y : 0 = -F_{N,C} - mg + F_{N,A}$$

c) Lagerreaktionen in Punkt  $B$  bestimmen,  $F_{N,C}$  sei bekannt

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



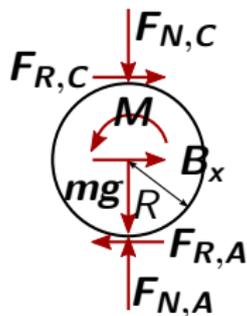
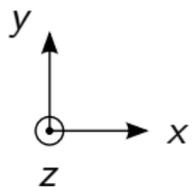
$$\sum F_x : 0 = B_x + F_{R,C} - F_{R,A}$$

$$\sum F_y : 0 = -F_{N,C} - mg + F_{N,A}$$

$$\Rightarrow B_x = \mu_R mg$$

c) Lagerreaktionen in Punkt  $B$  bestimmen,  $F_{N,C}$  sei bekannt

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\sum F_x : 0 = B_x + F_{R,C} - F_{R,A}$$

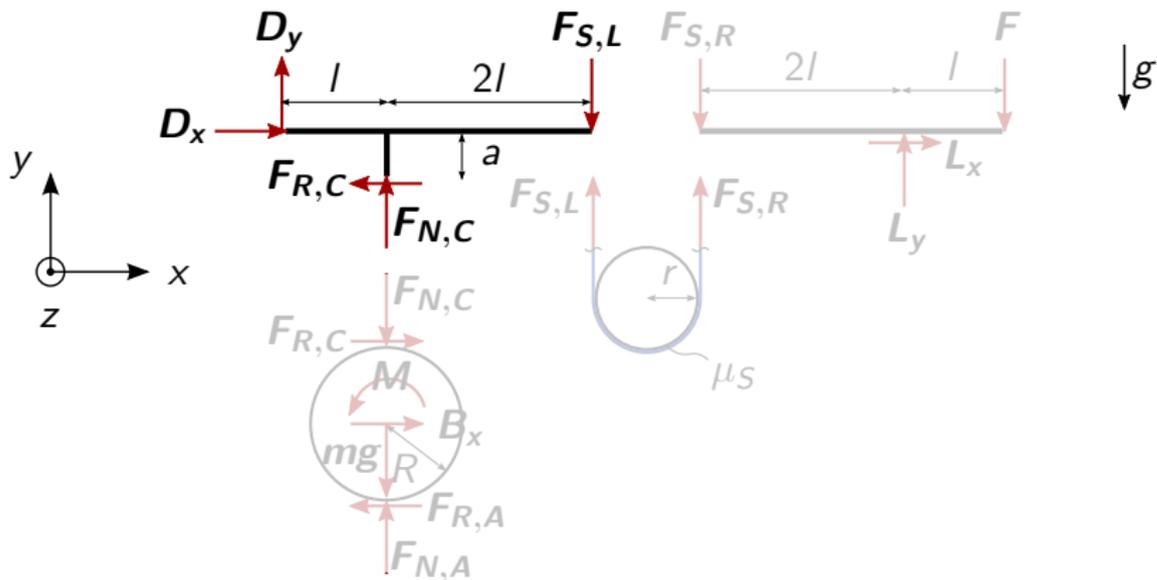
$$\sum F_y : 0 = -F_{N,C} - mg + F_{N,A}$$

$$\Rightarrow B_x = \mu_R mg$$

$$B_y = 0 \quad M^B = 0$$

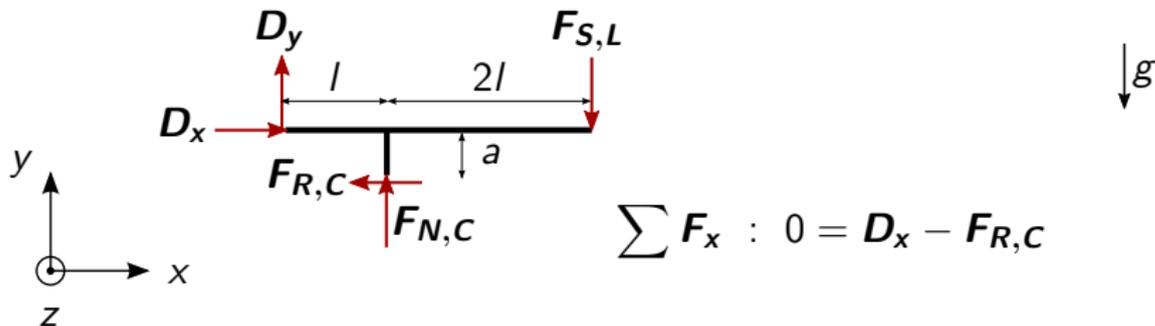
d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

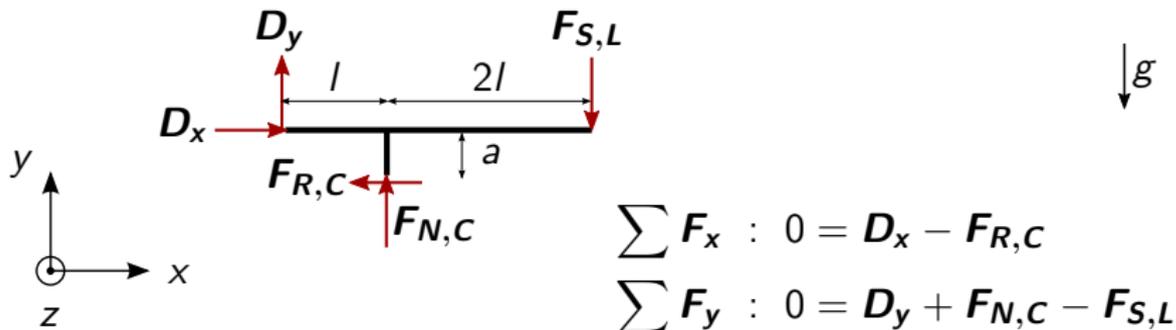
Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\sum F_x : 0 = D_x - F_{R,C}$$

d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$

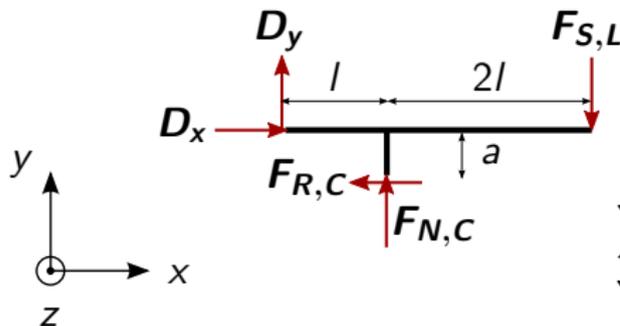


$$\sum F_x : 0 = D_x - F_{R,C}$$

$$\sum F_y : 0 = D_y + F_{N,C} - F_{S,L}$$

d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



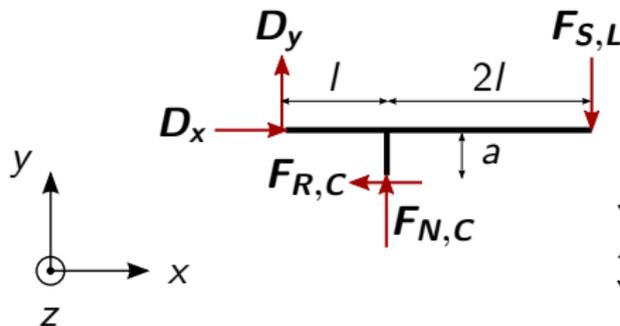
$$\sum F_x : 0 = D_x - F_{R,C}$$

$$\sum F_y : 0 = D_y + F_{N,C} - F_{S,L}$$

$$\sum M_z : 0 = -aF_{R,C} + lD_y - 2lF_{S,L}$$

d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\sum F_x : 0 = D_x - F_{R,C}$$

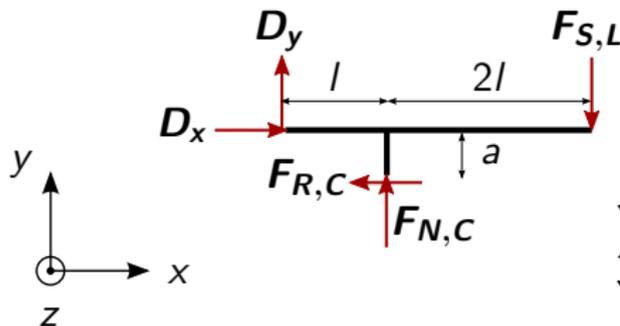
$$\sum F_y : 0 = D_y + F_{N,C} - F_{S,L}$$

$$\sum M_z : 0 = -aF_{R,C} + lD_y - 2lF_{S,L}$$

$$D_y = F_{N,C} \left( -\frac{a\mu_R}{3l} - \frac{2}{3} \right)$$

d) Lagerreaktionen in Punkt  $D$  und Seilkraft  $F_{S,L}$  bestimmen

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$\sum F_x : 0 = D_x - F_{R,C}$$

$$\sum F_y : 0 = D_y + F_{N,C} - F_{S,L}$$

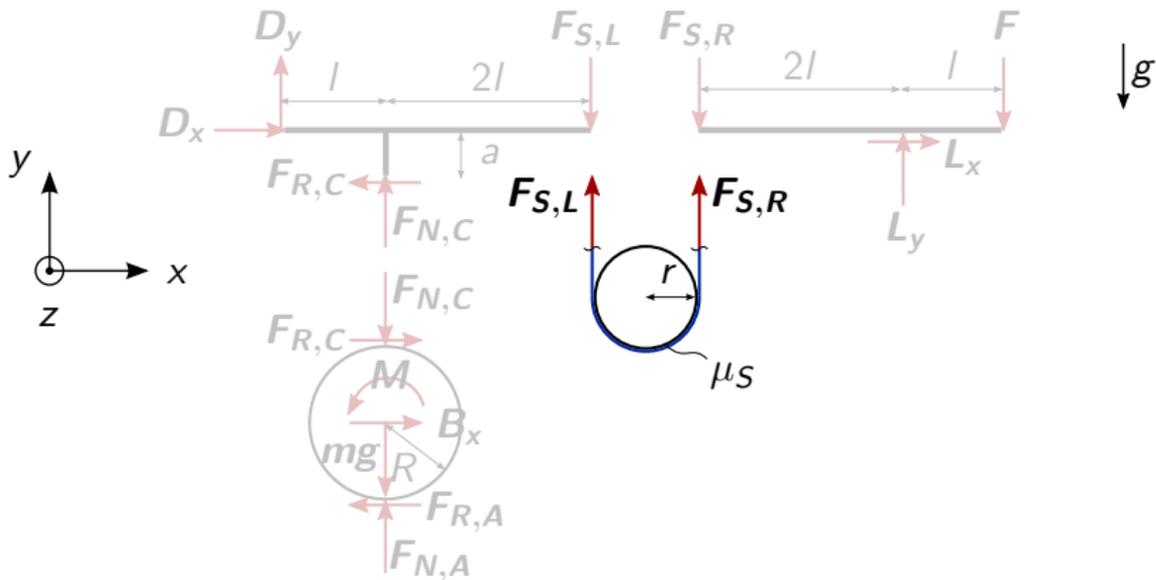
$$\sum M_z : 0 = -aF_{R,C} + lD_y - 2lF_{S,L}$$

$$D_y = F_{N,C} \left( -\frac{a\mu_R}{3l} - \frac{2}{3} \right)$$

$$F_{S,L} = \frac{F_{N,C}}{3} \left( 1 - \frac{a\mu_r}{l} \right)$$

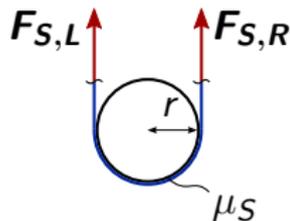
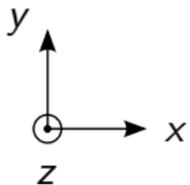
e) Seilkraft  $F_{S,R}$  bestimmen, um das Rad zu halten

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



e) Seilkraft  $F_{S,R}$  bestimmen, um das Rad zu halten

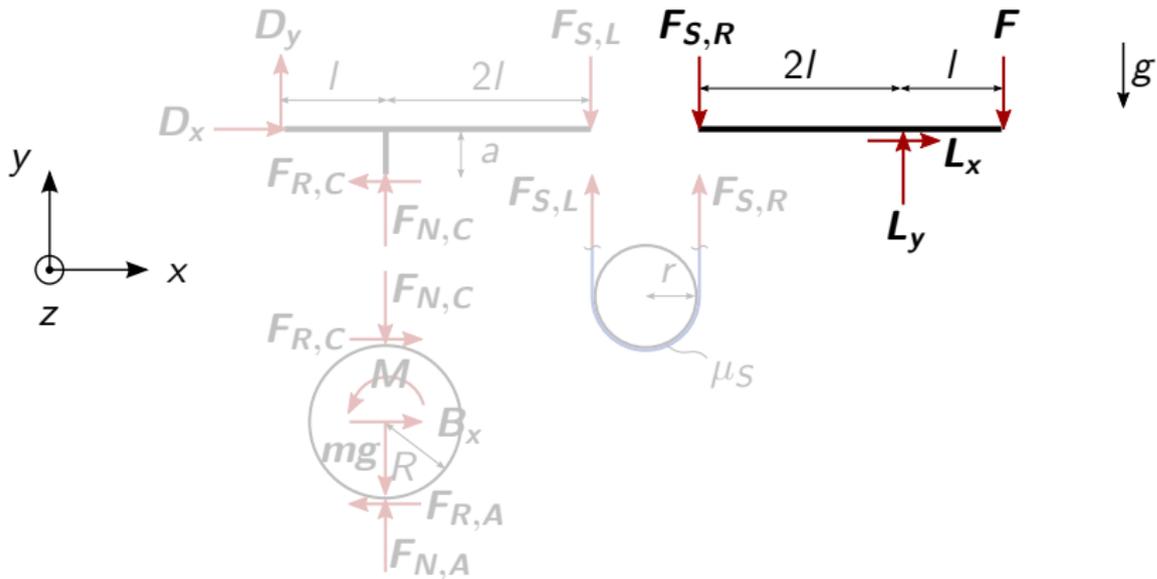
Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$



$$F_{S,R} = F_{S,L} e^{\pi \mu_S}$$

f) Betrag von  $F$  bestimmen, um das Rad zu halten

Gegeben:  $g$ ;  $m$ ;  $M$ ;  $R$ ;  $r$ ;  $l$ ;  $a$ ;  $\mu_S$ ;  $\mu_R$





Fragen?