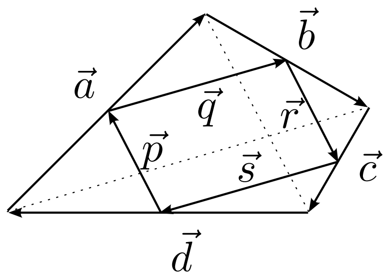
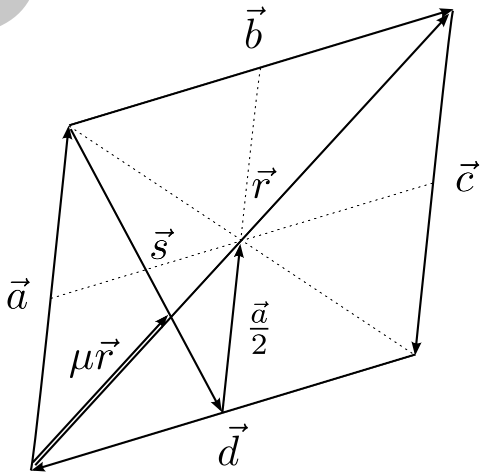
I.I.1



$$0 = \vec{a} + \vec{b} + \vec{c} + \vec{d} \quad \Leftrightarrow \quad \tfrac{1}{2}\vec{a} + \tfrac{1}{2}\vec{d} = -(\tfrac{1}{2}\vec{b} + \tfrac{1}{2}\vec{c})$$

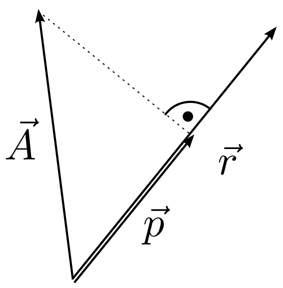
$$\Leftrightarrow \quad \vec{p} = -\vec{r} \quad \text{und} \quad \vec{q} = -\vec{s}$$

I.I.2



$$\frac{(\frac{1}{2}-\mu)|\vec{r}|}{\frac{|\vec{a}|}{2}} = \frac{\mu|\vec{r}|}{|\vec{a}|} \quad \Leftrightarrow \quad 3\mu = 1$$

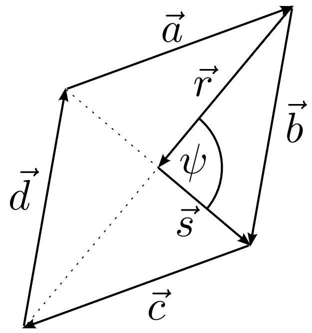
I.I.3



$$\vec{p} = \frac{\vec{A}\cdot\vec{r}}{|\vec{r}|^2} \cdot \vec{r} = \frac{1}{3} \begin{pmatrix} A_x + A_y + A_z \\ A_x + A_y + A_z \\ A_x + A_y + A_z \end{pmatrix}$$

$$|\vec{p}| = \sqrt{3 \cdot \tfrac{1}{9}(A_x + A_y + A_z)^2} = \tfrac{1}{\sqrt{3}}(A_x + A_y + A_z)$$

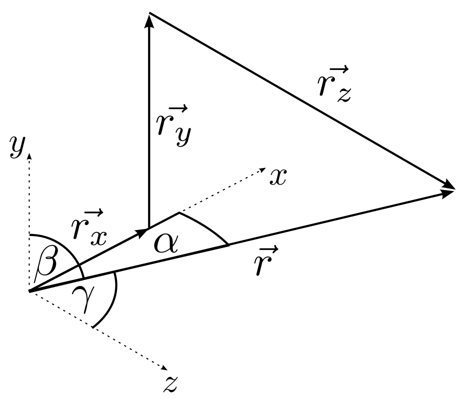
I.I.4



$$|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{d}| \quad \text{und} \quad \vec{a} = -\vec{d},\ \vec{b} = -\vec{c}$$

$$\cos(\psi) = \frac{\vec{r}\cdot\vec{s}}{|\vec{r}||\vec{s}|} = \frac{(\vec{b}+\vec{c})\cdot(\vec{b}+\vec{a})}{|\vec{b}+\vec{c}||\vec{b}+\vec{a}|} = \frac{\vec{b}^2-\vec{a}^2}{|\vec{b}-\vec{a}||\vec{b}+\vec{a}|} = 0$$

I.1.5

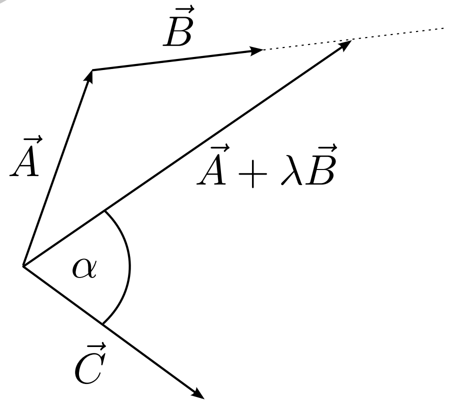


$$5 = \sqrt{(\tfrac{1}{6}l)^2 + (\tfrac{2}{6}l)^2 + (\tfrac{3}{6}l)^2} \quad \Leftrightarrow \quad \tfrac{1}{6}l = \tfrac{5}{\sqrt{14}}$$

$$\cos(\alpha) = \frac{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \frac{5}{\sqrt{14}} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \frac{5}{\sqrt{14}}\right| \cdot \left|\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right|}$$

$$\Leftrightarrow \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$$

I.1.6



$$\cos(60°) = \frac{1}{2} = \frac{\begin{pmatrix} 3 & + & \lambda \\ 6 & + & 2\lambda \\ 2 & - & \lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}}{\left|\begin{pmatrix} 3 & + & \lambda \\ 6 & + & 2\lambda \\ 2 & - & \lambda \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}\right|}$$

$$2\lambda^2 + 42\lambda + 33 = 0$$

I.1.7

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \qquad \text{und} \qquad \frac{\left|\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right|}{\left|\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}\right|} = \pm\frac{1}{\sqrt{7}}$$

I.1.8

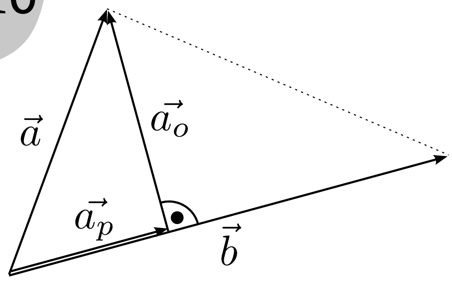
$$\vec{A}(\vec{B} \times \vec{C}) = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \begin{pmatrix} B_yC_z & - & B_zC_y \\ -B_xC_z & + & B_zC_x \\ B_xC_y & - & B_yC_x \end{pmatrix}$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$
$$= \vec{B}(\vec{C} \times \vec{A}) = \vec{C}(\vec{A} \times \vec{B})$$
$$= -\vec{A}(\vec{C} \times \vec{B}) = -\vec{B}(\vec{A} \times \vec{C}) = -\vec{C}(\vec{B} \times \vec{A})$$

I.1.9

$$\begin{pmatrix} 5 \\ \alpha \\ 2 \end{pmatrix} \left[\begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -10 \\ 4 \end{pmatrix} \right] = 14\alpha + 46$$

I.1.10



$$F_\Delta = \frac{1}{2}|\vec{a_o}||\vec{b}| = \frac{1}{2}|(\vec{a} - \frac{\vec{a}\vec{b}}{|\vec{b}|^2}\vec{b})||\vec{b}|$$

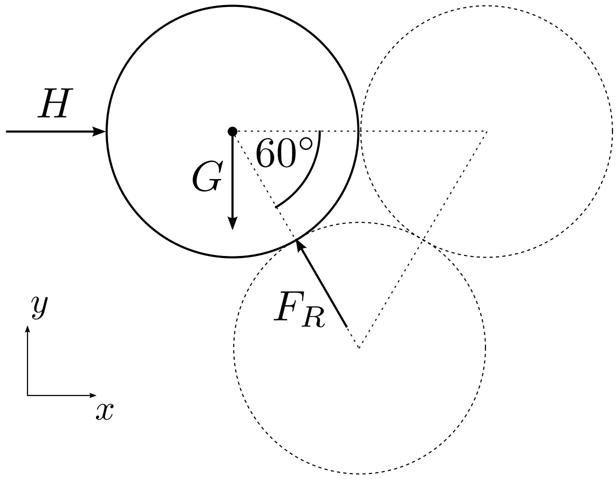
$$= \frac{1}{2}\left| \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} - \frac{34}{66}\begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix} \right| \left| \begin{pmatrix} 7 \\ 1 \\ 4 \end{pmatrix} \right|$$

I.1.11

$$\left[\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \right] \times \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \\ -4 \end{pmatrix}$$

I.2.1

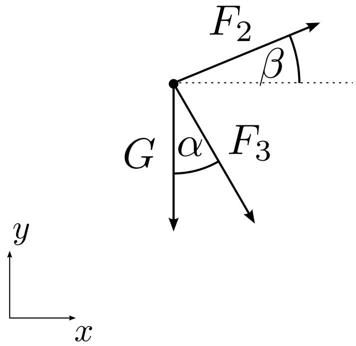


$$\sum \mathbf{F_x} \ : \ H - F_R \cos(60°) = 0$$

$$\sum \mathbf{F_y} \ : \ G - F_R \sin(60°) = 0$$

$$H = \frac{1}{\sqrt{3}} G$$

I.2.2



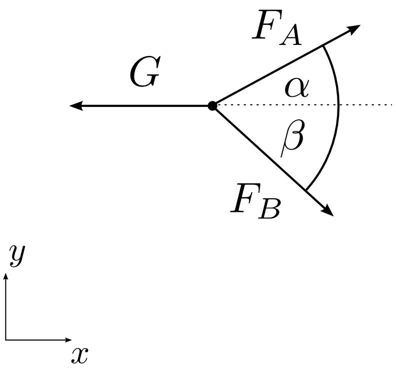
$$\sum \mathbf{F_x} \ : \ F_2 \cos(\beta) + F_3 \sin(\alpha) = 0$$

$$\sum \mathbf{F_y} \ : \ F_2 \sin(\beta) - F_3 \cos(\alpha) - G = 0$$

$$\tan(\beta) = \frac{a - l\cos(\alpha)}{l \sin(\alpha)}$$

$$F_2 = \frac{\sin(\alpha)}{\cos(\beta)} \frac{l}{a} G \ , \ F_3 = -\frac{l}{a} G$$

I.2.3



$$\sum \mathbf{F_x} \; : \; F_A \cos(\alpha) + F_B \cos(\beta) - G = 0$$

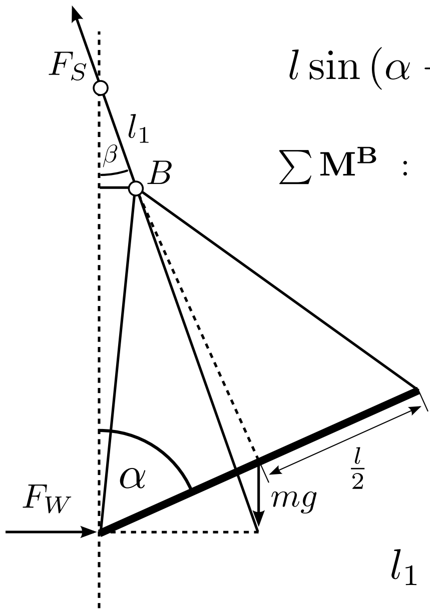
$$\sum \mathbf{F_y} \; : \; F_A \sin(\alpha) - F_B \sin(\beta) = 0$$

$$\tan(\alpha) = \tfrac{a}{l} \;,\; \tan(\beta) = \tfrac{b}{l}$$

$$F_A = \frac{1}{\cos(\alpha) + \frac{\sin(\alpha)}{\tan(\beta)}} G$$

$$F_B = \frac{a}{l \sin(\beta)(1 + \frac{a}{b})} G$$

I.2.4



$$l \sin(\alpha - 60°) = l_1 \sin(\beta) = l_1 \sqrt{1 + \frac{1}{\tan^2(\beta)}}$$

$$\sum \mathbf{M^B} \; : \; l \left(\tfrac{1}{2} \cos(\alpha) + \tfrac{\sqrt{3}}{2} \sin(\alpha) \right) \cdot F_W - l \left(\tfrac{1}{2} \sin(\alpha) - \tfrac{\sqrt{3}}{2} \cos(\alpha) \right) \cdot mg$$

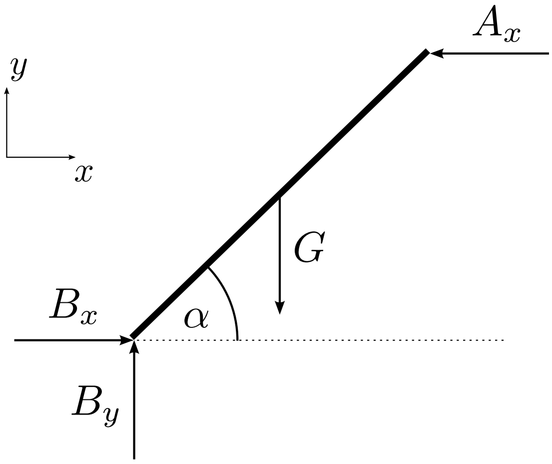
$$\frac{F_W}{\sin(\beta)} = \frac{m \cdot g}{\cos(\beta)}$$

$$\tan(\beta) = \frac{1}{\frac{1}{\sqrt{3}} + \tan(\alpha)}$$

$$l_1 = \frac{l}{2} \left(\sin(\alpha) - \sqrt{3} \cos(\alpha) \right) \cdot \sqrt{\left(\frac{1}{\sqrt{3}} + \tan(\alpha) \right)^2 + 1}$$

$$F_S = \frac{mg}{\cos(\beta)} = mg \sqrt{1 + \tan^2(\beta)} = mg \sqrt{1 + \frac{1}{\left(\frac{1}{\sqrt{3}} + \tan(\alpha) \right)^2}}$$

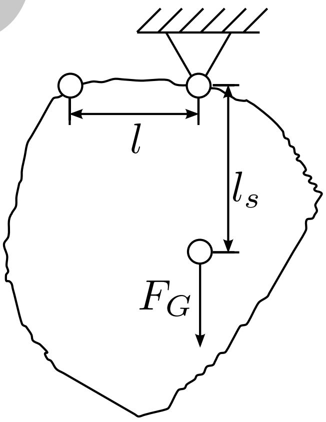
I.3.1



$$A_x = B_x \;,\; B_y = G$$

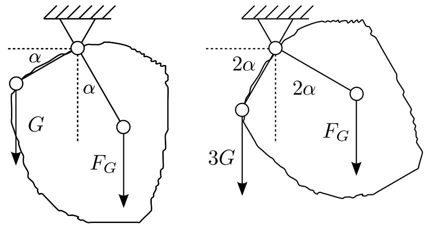
$$A_x = \frac{1}{2 \tan(\alpha)} G$$

I.3.2



$$\sum \mathbf{M_1} : l\cos(\alpha)\cdot G = l_s \sin(\alpha)\cdot F_G$$

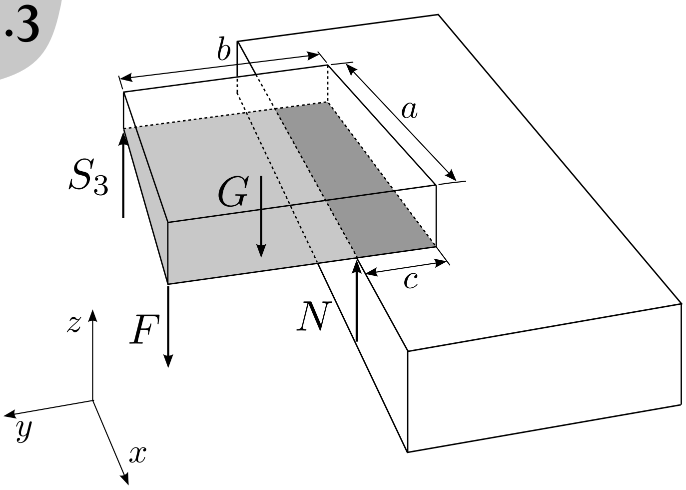
$$\sum \mathbf{M_2} : l\cos(2\alpha)\cdot 3G = l_s \sin(2\alpha)\cdot F_G$$



$$\frac{\cos(\alpha)}{\sin(\alpha)} = \frac{3}{2}\cdot\frac{\cos^2(\alpha)-\sin^2(\alpha)}{\sin(\alpha)\cos(\alpha)}$$

$$4\cos^2(\alpha) = 3$$

I.3.3

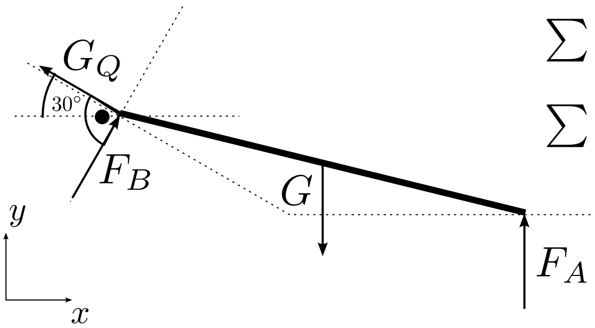


$$\sum \mathbf{M_x^{S_3}} : -\frac{b}{2}\cdot G + (b-c)\cdot N = 0$$

$$\sum \mathbf{M_y^{S_3}} : -\frac{a}{2}\cdot G + a\cdot N - a\cdot F = 0$$

$$N = \frac{b}{2(b-c)}G\ ,\ F = \frac{c}{2(b-c)}G$$

I.3.4

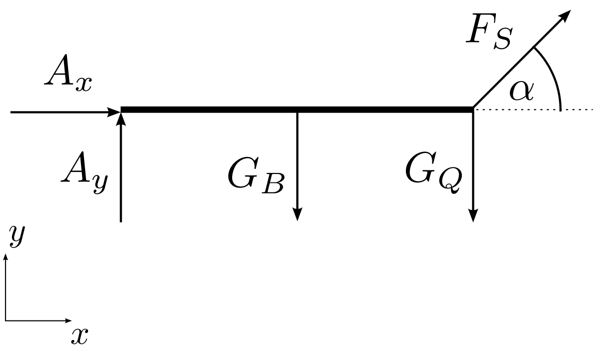


$$\sum \mathbf{F_x} : F_B\sin(30°) - G_Q\cos(30°) = 0$$

$$\sum \mathbf{F_y} : F_B\cos(30°) + G_Q\sin(30°) - G = 0$$

$$\sum \mathbf{M^B} : l\cdot F_A - \frac{l}{2}\cdot G = 0$$

I.3.5



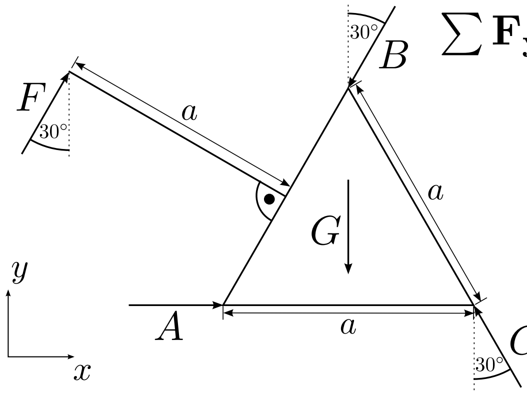
$$\sum \mathbf{F_x} : A_x + F_S\cos(\alpha) = 0$$

$$\sum \mathbf{F_y} : A_y - G_B - G_Q + F_S\sin(\alpha) = 0$$

$$\sum \mathbf{M^B} : \frac{l}{2}\cdot G_B - l\cdot A_y = 0$$

$$F_A = \sqrt{A_x^2 + A_y^2}$$

I.3.6



$$\sum \mathbf{F_x} \ : \ A + F\sin(30^\circ) - B\sin(30^\circ) - C\sin(30^\circ) = 0$$

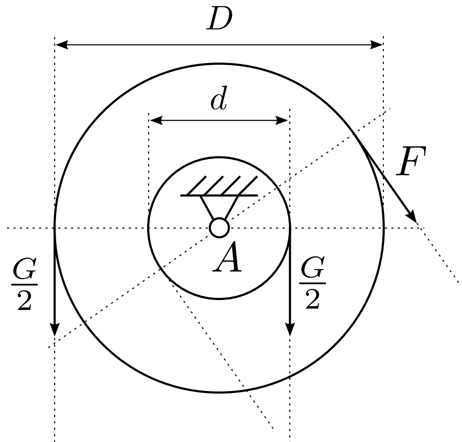
$$\sum \mathbf{F_y} \ : \ F\cos(30^\circ) - B\cos(30^\circ) - G + C\cos(30^\circ) = 0$$

$$\sum \mathbf{M^A} \ : \ -a\cdot F - \frac{a}{2}\cdot G + a\cos(30^\circ)\cdot C = 0$$

$$C = \frac{5}{2\sqrt{3}}F \ , \ B = (1+\frac{\sqrt{3}}{2})F$$

$$A = \frac{2}{\sqrt{3}}F$$

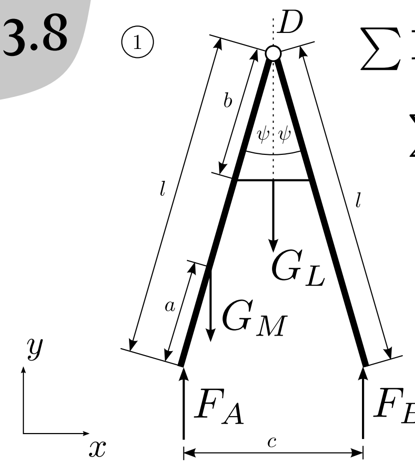
I.3.7

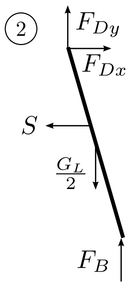


$$\sum \mathbf{M^A} \ : \ \frac{D}{2}\cdot\frac{G}{2} - \frac{d}{2}\cdot\frac{G}{2} - \frac{D}{2}\cdot F = 0$$

$$\Leftrightarrow F = \frac{D-d}{2D}G$$

I.3.8





$$\sum \mathbf{M_A^1} \ : \ -a\sin(\psi)\cdot G_M - l\sin(\psi)\cdot G_L + 2l\sin(\psi)\cdot F_B = 0$$

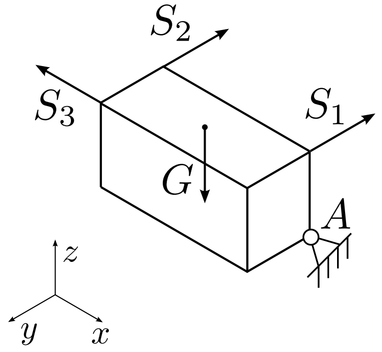
$$\sum \mathbf{M_D^2} \ : \ -b\cos(\psi)\cdot S - \frac{l}{2}\sin(\psi)\cdot\frac{G_L}{2} + l\sin(\psi)\cdot F_B = 0$$

$$\psi = \sin^{-1}\left(\frac{c}{2l}\right)$$

$$F_B = \frac{1}{2}(G_L + \frac{a}{l}G_M)$$

$$S = \frac{l\sin(\psi)}{b\cos(\psi)}(F_B - \frac{1}{4}G_L)$$

I.3.9

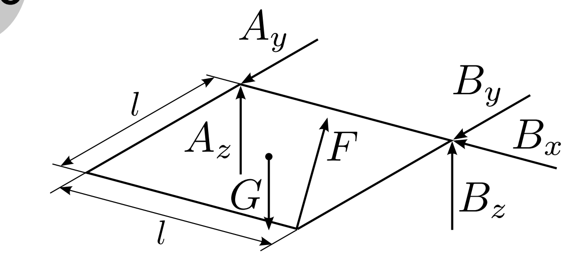


$$\sum \mathbf{M^A} \ : \ \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ -S_1 \\ 0 \end{pmatrix} + \begin{pmatrix} -3a \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ -S_2 \\ 0 \end{pmatrix}$$

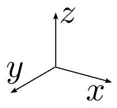
$$+ \begin{pmatrix} -3a \\ a \\ a \end{pmatrix} \times \begin{pmatrix} -S_3 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{3}{2}a \\ \frac{1}{2}a \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -G \end{pmatrix} = 0$$

$$\Leftrightarrow \begin{pmatrix} S_1 & = & 0 \\ S_3 & = & -\frac{3}{2}G \\ S_2 & = & \frac{1}{2}G \end{pmatrix}$$

I.3.10

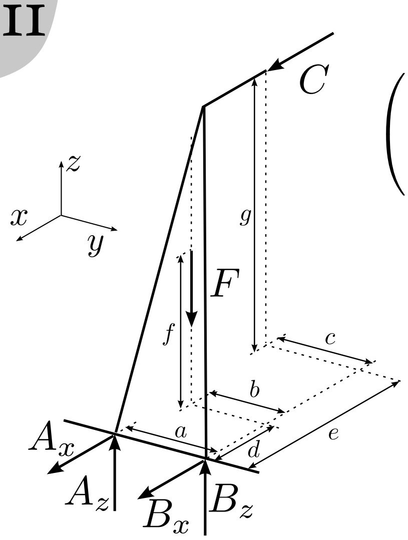


$$r_{F,norm} = \frac{2}{3}\begin{pmatrix} -1 \\ -\frac{1}{2} \\ 1 \end{pmatrix} \quad \Rightarrow \quad F = \begin{pmatrix} -\frac{2}{3}F \\ -\frac{1}{3}F \\ \frac{2}{3}F \end{pmatrix}$$



$$\begin{pmatrix} -l \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} -\frac{l}{2} \\ \frac{l}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -G \end{pmatrix} + \begin{pmatrix} 0 \\ l \\ 0 \end{pmatrix} \times \begin{pmatrix} -\frac{2}{3}F \\ -\frac{1}{3}F \\ \frac{2}{3}F \end{pmatrix} = 0$$

I.3.11



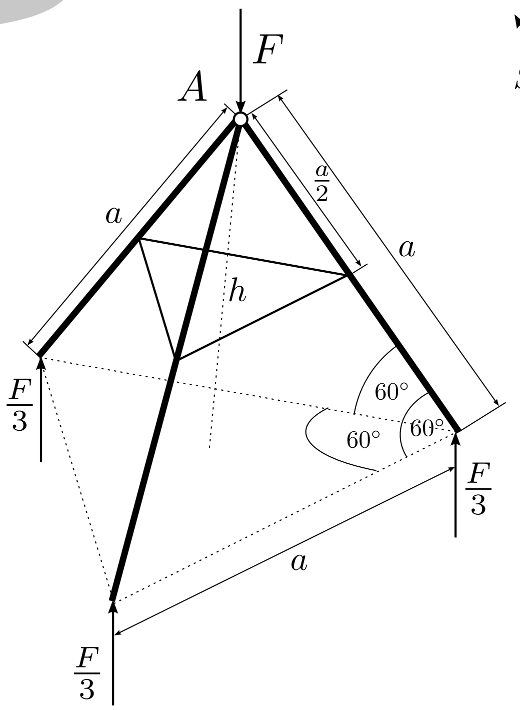
$$\begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} -d \\ -b \\ f \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} + \begin{pmatrix} -e \\ -c \\ g \end{pmatrix} \times \begin{pmatrix} C \\ 0 \\ 0 \end{pmatrix} = 0$$

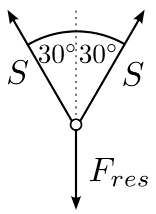
$$A_z = \frac{b}{a}F \ , \ C = \frac{d}{g}F \ , \ A_x = -\frac{cd}{ag}F$$

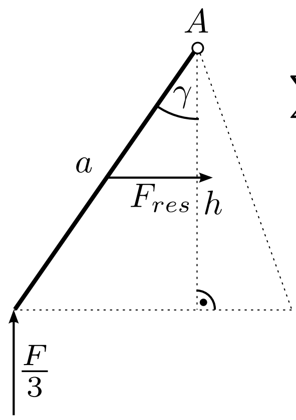
$$B_x = \frac{d}{g}\left(\frac{c}{a} - 1\right)F \ , \ B_z = \left(1 - \frac{b}{a}\right)F$$

$$A_z \ , \ B_z > 0 \quad \Leftrightarrow \quad 0 < b < a$$

I.3.12







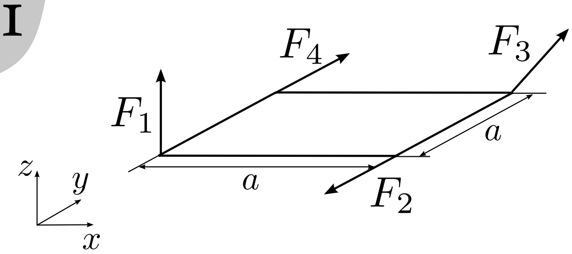
$$h = a\sqrt{\frac{2}{3}} \ \Rightarrow \ \sin(\gamma) = \frac{1}{\sqrt{3}}$$

$$\cos(\gamma) = \sqrt{\frac{2}{3}}$$

$$\sum \mathbf{M^A} \ : \ \frac{a}{\sqrt{6}} \cdot F_{res} - \frac{a}{\sqrt{3}} \cdot \frac{F}{3} = 0$$

$$S = \sqrt{\frac{2}{3}} \cdot \frac{F}{3}$$

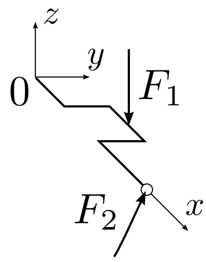
I.4.1



$$\vec{R} = \sum_i F_i$$

$$\sum \mathbf{M^A} \;:\; \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -F \\ 0 \end{pmatrix} + \begin{pmatrix} a \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} F \\ F \\ 0 \end{pmatrix} = \vec{M}_A$$

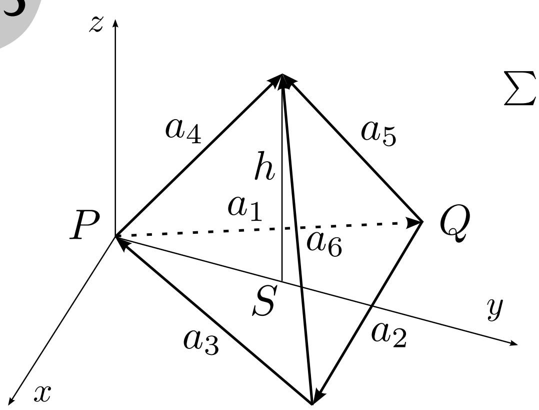
I.4.2



$$\vec{F} = \sum_i F_i$$

$$\sum \mathbf{M^A} \;:\; \begin{pmatrix} 12 \\ 10 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -100N \end{pmatrix} + \begin{pmatrix} 30 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -100N \\ 0 \\ 40N \end{pmatrix} = \vec{M}_0$$

I.4.3



$$\sum \mathbf{M^P} \;:\; \begin{pmatrix} 0 \\ \frac{\sqrt{3}}{2}A \\ 0 \end{pmatrix} \times \begin{pmatrix} A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}A \\ \frac{\sqrt{3}}{2}A \\ 0 \end{pmatrix} \times \begin{pmatrix} -\frac{1}{2}A \\ -\frac{1}{2\sqrt{3}}A \\ \sqrt{\frac{2}{3}}A \end{pmatrix}$$

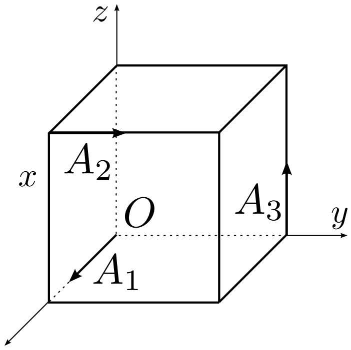
$$+ \begin{pmatrix} -\frac{1}{2}A \\ \frac{\sqrt{3}}{2}A \\ 0 \end{pmatrix} \times \begin{pmatrix} \frac{1}{2}A \\ -\frac{1}{2\sqrt{3}}A \\ \sqrt{\frac{2}{3}}A \end{pmatrix} = \vec{M}_P$$

$$\vec{R} = \vec{a_4} + \vec{a_5} + \vec{a_6} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{3}}A \\ \sqrt{\frac{2}{3}}A \end{pmatrix} + \begin{pmatrix} \frac{1}{2}A \\ -\frac{1}{2\sqrt{3}}A \\ \sqrt{\frac{2}{3}}A \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}A \\ -\frac{1}{2\sqrt{3}}A \\ \sqrt{\frac{2}{3}}A \end{pmatrix}$$

$$\vec{M}_Q = \vec{M}_P + \vec{r}_{QP} \times \vec{R}$$

Dyname (Vektorschraube): $\vec{M}_{Parallel} = \frac{\vec{R}\vec{M}_P}{|\vec{R}|^2} \cdot \vec{R} = -\frac{\sqrt{2}}{4}A \cdot \vec{R}$ und $\vec{r} = \frac{\vec{R} \times \vec{M}_P}{|\vec{R}|^2} + \mu \cdot \vec{R}$

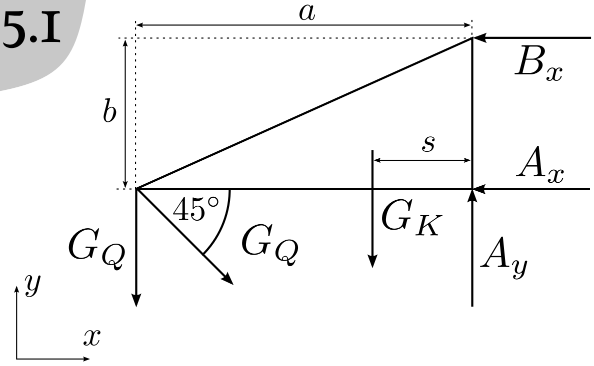
I.4.4



$$\vec{R} = \sum_i A_i \,,\; \vec{M} = \begin{pmatrix} a \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} 0 \\ A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ A \end{pmatrix}$$

$$\vec{M}_{Parallel} = \frac{\vec{R}\vec{M}}{|\vec{R}|^2} \cdot \vec{R} = \frac{a}{3} \cdot \vec{R} \quad \text{und} \quad \vec{r} = \frac{\vec{R} \times \vec{M}}{|\vec{R}|^2} + \mu \cdot \vec{R}$$

I.5.1

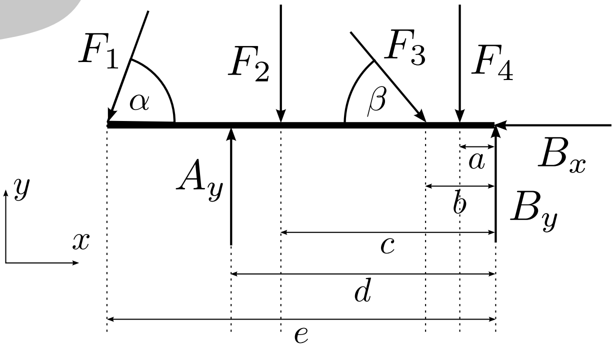


$$\sum \mathbf{F_x} \ : \ G_Q \cos(45°) - A_x - B_x = 0$$

$$\sum \mathbf{F_y} \ : \ -G_Q - G_Q \sin(45°) - G_K + A_y = 0$$

$$\sum \mathbf{M^B} \ : \ a \cdot G_Q + a \cdot G_Q \sin(45°) + s \cdot G_K + b \cdot B_x = 0$$

I.5.2



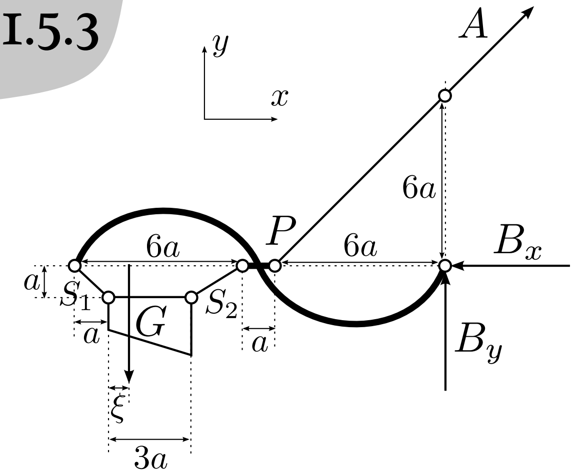
$$\sum \mathbf{F_x} \ : \ -F_1 \cos(\alpha) + F_3 \cos(\beta) - B_x = 0$$

$$\sum \mathbf{F_y} \ : \ -F_1 \sin(\alpha) + A_y - F_2 - F_3 \sin(\beta) - F_4 + B_y = 0$$

$$\sum \mathbf{M^B} \ : \ e \cdot F_1 \sin(\alpha) - d \cdot A_y + c \cdot F_2$$

$$+ b \cdot F_3 \sin(\beta) + a \cdot F_4 = 0$$

I.5.3



$$\sum \mathbf{F_x} \ : \ A\tfrac{\sqrt{2}}{2} - B_x = 0$$

$$\sum \mathbf{F_y} \ : \ A\tfrac{\sqrt{2}}{2} + B_y - G = 0$$

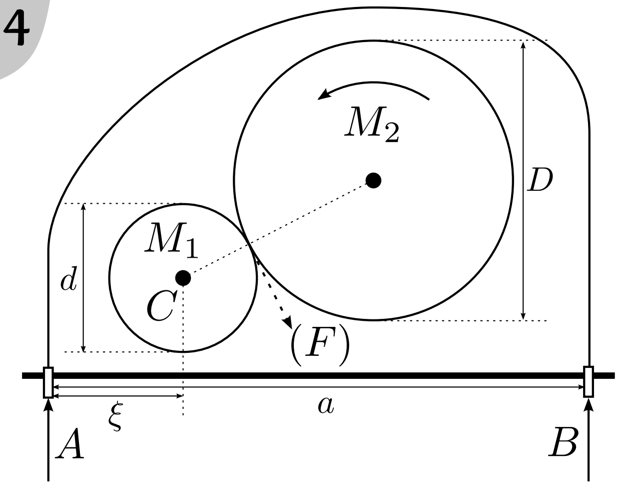
$$\sum \mathbf{M^P} \ : \ 6a \cdot B_y + (6a - \xi) \cdot G = 0$$

$$\xi \ : \ S_1 \cdot v = S_{2,O} + S_2 \cdot w$$

$$\Leftrightarrow \quad \begin{pmatrix} -a \\ a \end{pmatrix} v = \begin{pmatrix} 3a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} w$$

$$\Rightarrow \quad \xi = a$$

I.5.4



$$|A| = |B|$$

$$(F) = \frac{2|M_2|}{D} = \frac{2|M_1|}{d}$$

$$1: \quad \sum \mathbf{M^C} \ : \ -\tfrac{d}{2} \cdot F - M_2 - \left(\tfrac{d}{2} + \xi\right) \cdot A$$

$$+ \left(a - \tfrac{d}{2} - \xi\right) \cdot B = 0$$

$$\Rightarrow \Big|_{(A=-B)} \ : \ |A| = \frac{|M_1| + |M_2|}{a}$$

$$2: \quad \sum \mathbf{M^A} \ : \ M_2 + a \cdot B = 0$$